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Multifractal Detrended Cross-Correlation Analysis of Geochemical Element Concentration

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Abstract: We use multifractal detrended cross -fluctuation analysis (MF-DXA) to investigate nonlinear behavior of geochemical element concentration, Au-Cu-Pb-Zn-Ag, in Shangzhuang Deposit, Shandong Province, China. We find that the generalized Hurst exponent $h(q)$ and cross-correlation exponent $h_{xy}(q)$ decrease with the increase of q, which indicate that all element concentration series and their cross pairs exhibit multifractal phenomena. By comparing the variability of $h(q)$ and $h_{xy}(q)$, we have found that the multifractal behavior is more obvious when $q > 0$ than $q < 0$ for the element Au-Cu-Pb-Zn and their cross pairs. These analyses, given quantitative information about the complexity of the element concentration, lead to a better understanding of the geochemical phenomena underlying mineralization process.

Keywords: Cross-correlation, detrended fluctuation analysis, geochemical element, multifractal, nonlinear.

1. INTRODUCTION

Tempo-spatial distribution characteristics of geochemical element distribution is the basis of metallogenic prognosis for a region. In recent decades, the distributions of geochemical elements in rocks has been intensively investigation and research to find out universal law [1]. Geochemical element distribution often shows a highly irregular structure, and exhibits scale-dependent changes in structure, and needs nonconventional statistical methods. For a better comprehension of the element grade of volatility function properties, it is necessary to ascertain the element concentration changes by investigating the structure of latency at the microscopic level [2,3]. Nonlinear properties of geochemical anomalies distribution have been observed in different media caused by underlying mineralization in different ways [4, 5]. Cheng proposed that geological anomalies are usually relevant to geochemical and geophysical anomalies because of their distinct physical and chemical properties and their surroundings [6]. The distributions of geochemical elements in disseminated-veinlet gold deposit exhibit fractal and multifractal characteristics [7-9]. But, in many cases, there still exists cross correlation between different geochemical elements. Looking for long-range correlations in geochemical element concentration series and for the cross-correlations between different elements concentration sets by multifractal analysis would help to promote our understanding of the corresponding dynamics and their future evolution.

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In order to find the multifractal features of nonstationary series, Kantelhardt et. al proposed the method of multifractal detrended fluctuation analysis (MF-DFA) [10]. Recently, a method to unveil the multifractal features of two cross-correlated signals and higher-dimensional multifractal measures was introduced as the generalization of DCCA [11]. The higher-dimensional method was named multifractal detrended cross-fluctuation analysis (MF-DXA) [12], which has been widely used to analysing data in different fields, such as financial, sunspot and river flow fluctuations, and meteorological [13-16].

In this paper, we used MF-DXA to investigate the correlation between Au and Cu-Pb-Zn-Ag in Shangzhuang Deposit, Shandong Province, China. Some interesting and valuable phenomena have been found.

2. MF-DXA

Firstly, we briefly introduce the MF-DXA [11, 12]. We suppose that there exist two tempo-spatial series $\{x_i\}$ and $\{y_i\}$, i=1, 2, …., N, where N is the series length. Then, cumulative sum is determined for series $\{x_i\}$ and $\{y_i\}$ respectively:

$$X(i)=\sum_{k=1}^{i}[x_k-\langle x_k\rangle],\ Y(i)=\sum_{k=1}^{i}[y_k-\langle y_k\rangle] \tag{1}$$

where $\langle\bullet\rangle$ represents the average value. The entire series were divided into Ns = int(N/s) over-lapping boxes. The length of the boxes is s. Local trends are evaluated by means of the m-th order polynomial fitting to each of segments. Then the detrended covariance is given by

$$f^2(s,v) = \frac{1}{s}\sum_{i=1}^{s} | X_v(i) - \tilde{X}_v(i) || Y_v(i) - \tilde{Y}_v(i) | \quad (2)$$

where $\tilde{X}_v(i)$ and $\tilde{Y}_v(i)$ are the fitting polynomials in segment v. Over all segments are averaged to gain q-th order detrended covariance, defined by:

$$F_{xy}(s,q) = \{\frac{1}{N_s}\sum_{v=1}^{N_s}[f^2(s,v)]^{q/2}\}^{1/q} \quad (3)$$

for $q \neq 0$

when q=0

$$F_0(s,0) = \lim_{q\to 0} F_{xy}(s,q) = \exp\{\frac{1}{2N_s}\sum_{v=1}^{2N_s}[\ln f^2(s,v)]\} \quad (4)$$

For several scales s, Eqs. (2) - (4) need to be repeated. If series $\{x_i\}$ and series $\{y_i\}$ are correlated by long-range power law, then scaling relation between detrended fluctuation function $F_{xy}(s,q)$ and the size scale s can be decided as

$$F_{xy}(s,q) \propto s^{h_{xy}(q)} \quad (5)$$

where $h_{xy}(q)$ is cross-correlation exponent. The $h_{xy}(q)$ can be estimated by log-log plots of $F_{xy}(s,q)$ *versus* s for each value of q analysis. When q = 2, the MF-DXA is the standard DCCA. When $\{x_i\}$ = $\{y_i\}$, MF-DXA reduces to the classic MF-DFA, $h_{xy}(q) = h_{xx}(q) = h(q)$. Generally, $h_{xy}(q)$ depends on q, which indicates the behavior of multifractality. The multifractality degree is able to be quantified by $\Delta h = h_{\max}(q) - h_{\min}(q)$.

3. RESULTS OF ANALYSIS

3.1. Data Acquisition and Descriptive Statistics

Shangzhuang gold deposit is located at Wangershan fault zone of northwestern of Shandong province, China. The ore bodies occur at the hanging wall and footwall of Wangershan fault in vein irregular shapes with the NE strike NW trend, 30°-60° in dip and the lateral trending of SW. In the field, main rock types are plagioclase-metamorphic gneiss, amphibolites and granulite in Jiaodong group, and Linglong and Guojialing granites. the method of Drilling and Control Source Audio-frequency Magnetotelluric Detecting shows that Shangzhuang gold deposit is a vertical two layer structure. The upper layer is the Linglong granite. The lower is the Guojialing granite. The main ore zone is of 1800m long, 2～4m thick, and around 750m depth [17, 18].

The composition of alteration rock in the central of alteration belt has the largest degree of variation, a large number of permeability fluids carry active components into the dilation space, which has significance to the further enrichment of the ore-forming element(Au) and the remobilization of correlative elements (Cu- Zn- Pb- Ag).

The data of geochemical element concentration, Au-Cu-Pb-Zn-Ag, were obtained from the continuous channels with 1m length in Shangzhuang Deposit. These samples were assayed. The results are used for reserve calculation as well as for this study.

In order to describe dominant features of data of geochemical element concentration, a statistical analysis was performed.

We performed a Jarque-Bera (JB) test for normality. The JB statistical quantity is

$$JB = n\left[\frac{S^2}{6} + \frac{(K-3)^2}{24}\right] \quad (6)$$

where s is skewness, k is kurtosis, and n is the sample data length. Main descriptive statistics results of the geochemical element concentration, Au-Cu-Pb-Zn-Ag, are shown in Table **1**.

According to the data in Table **1**, the kurtosis values greater than 3 are display aiguille in the probability distribution. The skewness values greater than 0 are display positive skewness. The results of Jarque-Bera test show that the values of statistic are more than the significance at the 0.05 level, which indicates sample distribution departure from Gaussian normality. So we should reject null hypothesis of following a normal distribution to five samples.

Table1. Results obtained from the statistical analysis of geochemical element concentration.

(1-1)

Element	Sample Interval	Mean (g/t)	Standard Deviation (g/t)
Au	[0.10,15.00]	1.0818	1.3605
Cu	[5.00,138.60]	8.8466	8.3782
Pb	[5.70,180.80]	23.5422	13.0665
Zn	[5.50,202.10]	30.7875	15.9711
Ag	[0.05,0.36]	0.0805	0.0256

(1-2)

Element	Kurtosis	Skewness	Jarque-Bera Statistic
Au	48.6248	6.0647	55148.1812*
Cu	148.5691	11.2145	130553.1177*
Pb	76.4542	7.0725	54865.7166*
Zn	29.6186	3.687	20495.9535*
Ag	35.7305	4.3061	29718.4715*

Note: *Statistical significance at the 5% level.

3.2. Cross-Correlation Test

We used a new cross-correlation test, proposed by Podobnik *et al*. [19], to quantify bivariate cross-correlation between elements Au and Cu-Pb-Ag. We suppose that there exist two discrete-series $\{x_i\}$ and $\{y_i\}$, sharing the same length N, and no cross correlations between them. We define the cross-correlation function:

$$C_k = \frac{\sum_{i=k+1}^{N} x_i y_{i-k}}{\sqrt{\sum_{i=1}^{N} x_i^2}\sqrt{\sum_{i=1}^{N} y_i^2}} \tag{7}$$

The cross-correlation coefficient C_k is a normal distribution for asymptotically large values of *N*. Then $\frac{C_k}{\sqrt{(N-k)/N^2}}$ asymptotically behaves as a standard normal distribution, with zero mean and unit variance. The sum of squares of these variables approximately follows a chi-square distribution (χ^2 -distribution). According to the definition of χ^2 -distribution, we can obtain the cross-correlations statistic

$$Q_{cc}(m) = N^2 \sum_{k=1}^{m} \frac{C_k^2}{N-k} \sim \chi^2(m) \tag{8}$$

which is approximately $\chi^2(m)$ distributed, where m is a degrees of freedom. If the cross-correlations test values are larger than the critical value of $\chi^2(m)$, then the cross-correlations are significant. For various values *m,* we used the critical values of χ^2 -distribution at the 0.05 significance level.

For each pair of the cross original data $\{x_i\}$ and $\{y_i\}$, $Au \cdot Cu$, $Au \cdot Pb$, $Au \cdot Zn$ and $Au \cdot Ag$, $Q_{cc}(m)$ were computed, with *m* = 1*,* 2*, ...,* 500, using Eq. 7. All the $Q_{cc}(m)$ of $Au \cdot Cu$, $Au \cdot Pb$, $Au \cdot Zn$ and $Au \cdot Ag$, are larger than the critical values of $\chi^2(m)$ distribution at the 0.05 significance level (see Fig. **1**). The difference between $Q_{cc}(m)$ and critical values can describe strength of cross-correlations. However, the $Q_{cc}(m)$ test is only qualitative. Then, we applied MF-DXA method to affirm results obtained above.

3.3. MF-DXA Analysis of Geochemical Element Concentration

For $Au \cdot Cu$, $Au \cdot Pb$, $Au \cdot Zn$ and $Au \cdot Ag$, $F_{xy}(s)$ are calculated by Eqs. (1) - (5) with *q* ranging from -5 to 5 in increments of 0.5 and *m*=2. In order to avoid the statistical error dependent on size *s*, we take *s* from 10 to int (*N*/5). Fig. (**2**) shows the fluctuations $F_{xy}(s)$ *versus s* for different values of *q* (*q*=-5, -2, 0, 2, 5) for three sample. The cross-correlation exponent $h_{xy}(q)$ can be obtained by observing the slope of plot $(\log(F_{xy}(s)), \log(s))$.

In Fig. (**3**), the relationship between cross-correlation exponent $h_{xy}(q)$ and q by MF-DXA is displayed. For comparison, we also estimate the generalized Hurst exponents *h*(*q*) of time series $\{x_i\}$ and $\{y_i\}$ by MF-DFA. When *q* = 2, Hurst exponent describes the persistence of autocorrelation in a separately analyzed time series. If Hurst exponent $h(2) > 0.5$, the system exhibits persistent properties; if $h(2) < 0.5$, it is anti-persistent. But for the cross-correlation exponent. Table2 shows that all of the $h(2)$ are greater than 0.5, and the $h(2)$ of Zn is the largest, which indicate that the geochemical elements concentration have the long-range correlation and element Zn has stronger than other elements. For all cross element pairs, $h(q)$ and $h_{xy}(q)$ are decreasing functions and exhibit a strong dependence on *q* (see Fig. **3**), which confirm the multifractal behavior of these series. Apparently, stronger multifractality corresponds to higher variability of $h(q)$. Moreover, the exponent Δh_{xy} is less when $q < 0$ than $q > 0$, where $\Delta h_{(q<0)} = h_{\max}(q) - h(0)$, $\Delta h_{(q>0)} = h(0) - h_{\min}(q)$. Thus, the multifractal behavior is more obvious when $q > 0$ than $q < 0$ for the elements Au-Cu-Pb-Zn-Ag, and their cross pairs.

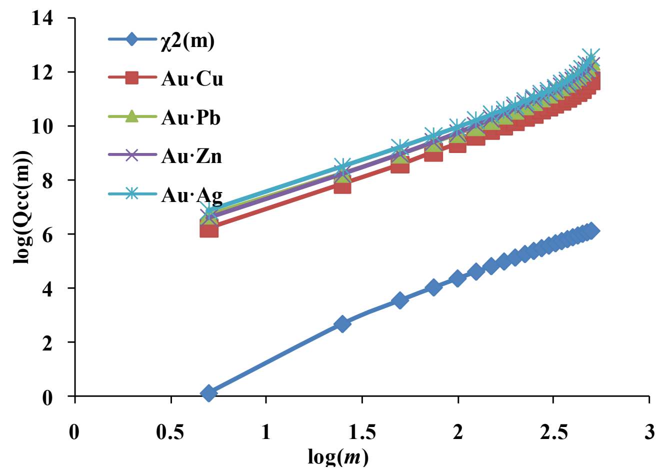

Fig. (1). $Q_{cc}(m)$ versus the degrees of freedom *m* for the Au·Cu, Au·Pb, Au·Zn and Au·Ag pairs, and for $\chi^2(m)$.

Table 2. The result of generalized Hurst exponents and cross-correlation exponents.

(2-1)

Element	h (2)	$\Delta h_{xx(q<0)}$	$\Delta h_{xx(q>0)}$
Au	0.53	0.24	0.42
Cu	0.62	0.31	0.87
Pb	0.66	0.45	0.65
Zn	0.80	0.11	0.26
Ag	0.68	0.13	0.37

(2-2)

Cross-Element	$h_{xy}(2)$	$\Delta h_{xy(q<0)}$	$\Delta h_{xy(q>0)}$
Au·Cu	0.73	0.20	0.53
Au·Pb	0.66	0.23	0.51
Au·Zn	0.71	0.15	0.30
Au·Ag	0.70	0.12	0.25

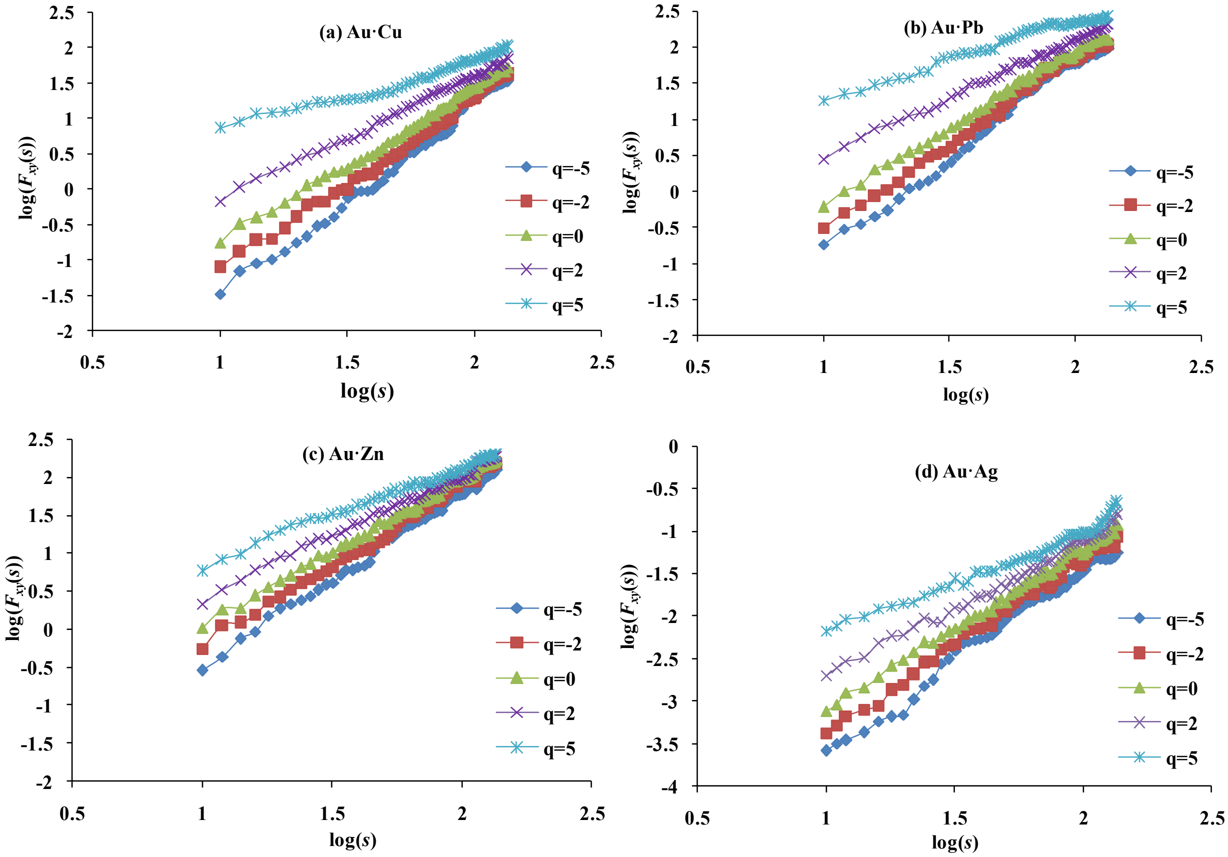

Fig. (2). The nonlinear relationships between log $F_{xy}(s)$ and log(s), m=2. (**a**) Au·Cu; (**b**) Au·Pb; (c) Au·Zn and (**d**) Au·Ag.

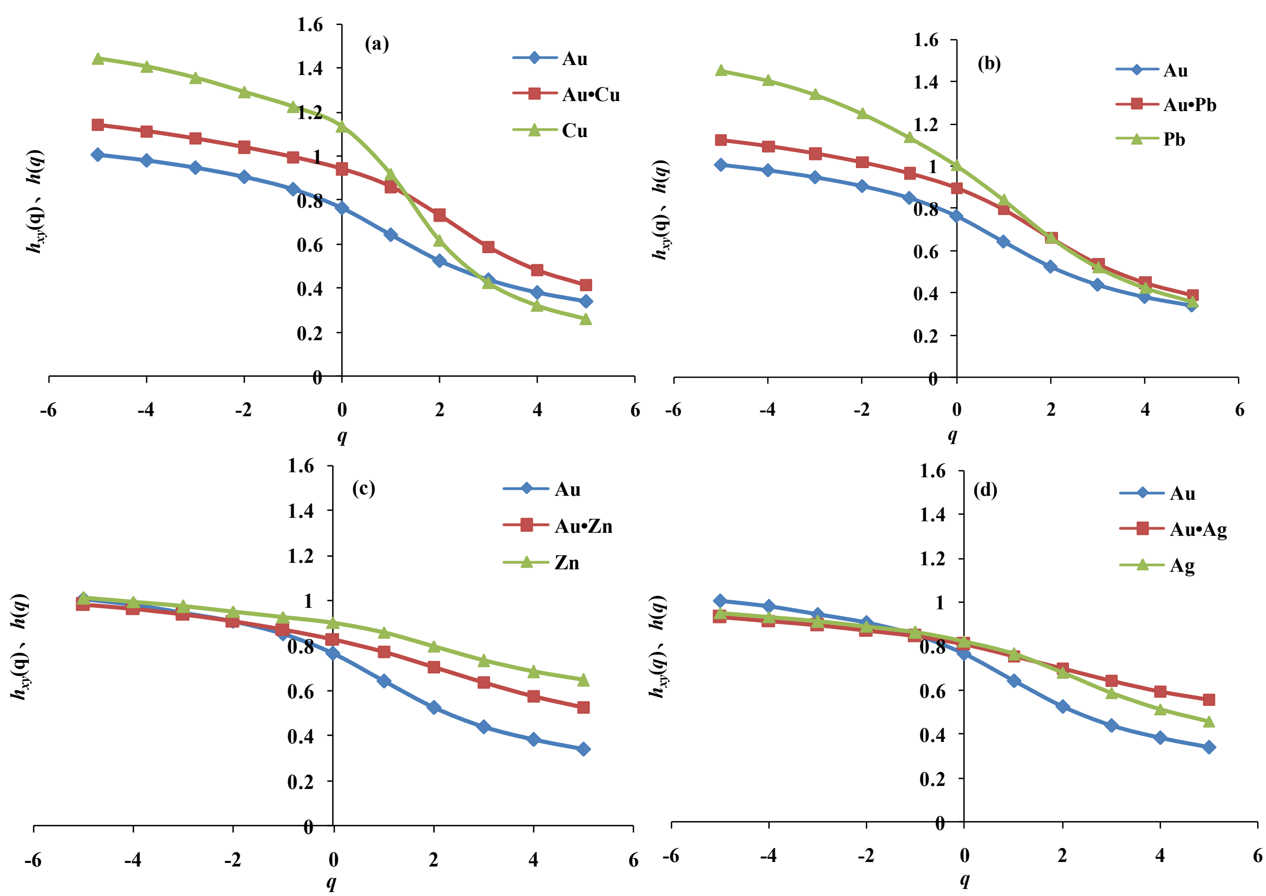

Fig. (3). Generalized scaling exponent $h(q)$ and cross-correlation exponent $h_{xy}(q)$.

CONCLUSION

Identifying the mechanisms and interactions that influence the spatial structure of geochemical element concentration distribution is important for both metallogenic prognosis and quantitative assessment. In the paper, we have analyzed not only qualitatively for cross-correlation, but also quantitatively for the multifractal behavior of geochemical element concentration of Au-Cu-Pb-Zn-Ag, in Shangzhuang Deposit, Shandong Province, China, by MF-DXA technique. The results show that the generalized Hurst exponent $h(q)$ and cross-correlation exponent $h_{xy}(q)$ decrease with the increase of q, which indicate that all element concentration series and their cross pairs exhibit multifractal phenomena. By comparing the variability of $h(q)$ and $h_{xy}(q)$, we have found that the multifractal behavior is more obvious when $q > 0$ than $q < 0$ for the elements Au-Cu-Pb-Zn-Ag and their cross pairs. These analyses, given quantitative information about the complexity of the element concentration series, lead to a better understanding of geochemical phenomena underlying mineralization process.

CONFLICT OF INTEREST

The author confirms that this article content has no conflict of interest.

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