38 *The Open Materials Science Journal*, 2010, *4*, 38-63

Hardness and Case Depth Analysis Through Optimization Techniques in Surface Hardening Processes

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Abstract: Surface engineering and surface engineered materials find wide applications in engineering industries in recent years. Inconsistency in hardness and case depth has resulted in the further optimization of the process variables involved in surface hardening. In the present study, the following operating parameters viz. preheating, carbon potential, holding position, furnace temperature, carburising time, quenching medium, quenching temperature, quenching time, tempering temperature and tempering time were taken for optimization using the Taguchi and Factorial design of experiment concepts. From the experiments and optimization analysis conducted on EN29 and EN34 materials it was observed that furnace temperature and quenching time had equal influence in obtaining a better surface integrity of the case hardened components using gas carburizing. Preheating before gas carburizing further enhanced the surface hardness and the depth of hardness. In the case of induction hardening process, power potential played a vital role in optimizing the surface hardness and the depth of hardness.

Keywords: Pinion, hardness, case depth, taguchi techniques, optimization, process variables.

1. INTRODUCTION

Changing demands of dynamic market place have improved and increased the commitment to quality consciousness. All over the world, companies are developing quality management systems like ISO 9001-2000 and investing in total quality [1]. One of the critical requirements for the ISO 9001-2000 is adequate control over process parameters. An auditing report of the ISO indicates that the majority of the heat treatment processes in industries present improper application of process variables and inadequate control over the process parameters [2]. Adequate control of process variables is possible if the level at which each of the parameters has to be maintained. Optimization is one of the approaches that help in finding out the right level or value of the parameters that have to be maintained for obtaining quality output. Determination of optimum parameters lies in the proper selection and introduction of suitable design of experiment at the earliest stage of the process and product development cycles so as to result in the quality and productivity improvement with cost effectiveness [3].

Investigations indicate that in surface hardening processes Heat treatment temperature, rate of heating and cooling, heat treatment period, Quenching media and temperature [4], Post heat treatment and pre-heat treatment processes are the major influential parameters, which affect the quality of the part surface hardened. This paper deals with the optimization studies conducted to evaluate the effect of various process variables in Gas Carburizing and Induction Hardening on the attainable Hardness and Case depth [5].

In this study, Taguchi's Design of Experiment concept has been used for the optimization of the process variables of Gas Carburizing process and Factorial design of Experiment for the optimization of process variables of Induction Hardening process. Taguchi's L27 orthogonal array and 3^3 Factorial array have been adopted to conduct experiments in Gas Carburizing and Induction Hardening processes respectively.

Optimum heat treatment conditions have been arrived by employing higher hardness cum case depth are better as the strategies. The secondary objectives of identifying the major influential variables on Hardness and Case depth have also been achieved.

2. GAS CARBURIZING PROCESS VARIABLES OPTIMIZATION USING TAGUCHI'S METHOD

In this study, Taguchi's L27 orthogonal array of Design of Experiment is used for the optimization of process variables of Gas carburizing process to improve the surface hardness and case depth of a Case hardened component. All these experiments were carried out by Repetition Method. Two different optimization analyses (Response Graph Analysis and Signal to noise Ratio analysis) have been done on the materials selected for the study. The materials used were EN 29 and EN 34. Experiments have been conducted on the machined component pinion of steering wheel assembly.

The normal procedure followed in converting the raw material into a finished product is shown in Fig. (**1**).

3. HARDNESS OF PINION

Hardness of a material is generally defined as resistance to permanent indentation under static or dynamic loads. Engineering materials are subjected to various applications where the load conditions and functional requirements may vary widely [6].

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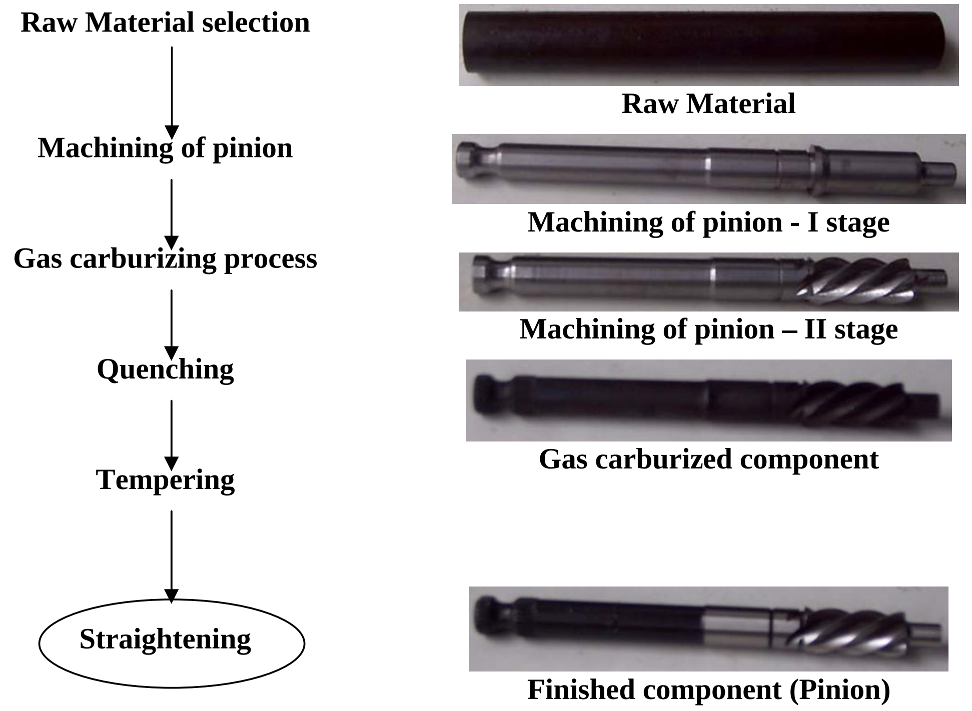


Fig. (1). Sequence of operations in gas carburising process.

In automobiles, power steering is an important assembly in which Rack and pinion are the major components subjected to twisting load. In order to improve the wear resistance characteristics and have high reliability, the components (Rack and Pinion) are subjected to case hardening.

The major problem in case hardening is inconsistency in hardness and case depth obtained. The magnitude of hardness depends on the process variables of any surface hardening process. Hence, in the present research, process variable optimization study has been carried for obtaining higher surface hardness on the pinion material (Fig. **2**) used in the power steering assembly of the automobile.

3.1. Hardness Optimization – Response Graph Method

Response graph method gives the output of interest to be optimized i.e., minimize, maximize, targeted, etc. The output can be more that one and also it can be quantitative or qualitative [7, 8]. The conditions underwhich gas carburizing experiments have conducted are given in Table **1**. Gas carburizing of selected materials have been done in a Unitherm Gas Carburizing Furnace (Fig. **3**) where Methanol along with Acetone is used as carburizing medium. The specifications of Gas carburizing furnace and operating conditions with range are given in Table **2**. The test results are reported in Tables **3** and **4**.


Before Carburizing

After Carburizing

Fig. (2). Pinion used in the power steering assembly of the automobile.

Table 1. Gas Carburizing-Operating Conditions

S. No.	Factors	Notation	Level 1	Level 2	Level 3
1	Furnace temperature	A	870°C	910°C	940°C
2	Quenching Time	B	60 minutes	90 minutes	120 minutes
3	Tempering Temperature	C	150°C	200°C	250°C
4	Tempering Time	D	80 minutes	100 minutes	120 minutes
5	Preheating	E	No preheating	150°C	No preheating

Fig. (3). Gas carburizing furnace.

The experiments have been conducted based on L27 orthogonal array system proposed in Taguchi's mixed level series DOE with interactions as given below:

i) Furnace Temperature *vs* Quenching time (AxB)

ii) Furnace Temperature *vs* Tempering Temperature (AxC)

Table 2. Specifications and Operating Conditions of Gas Carburizing

Material Used : AISI – Low Carbon steel materials
Diameter : 17.3 mm ; Length : 150 mm
Furnace Details:
Methanol – Acetone Unitherm Gas Carburizing Furnace of 3 ½ m depth
Electrical rating : 130 KW
Temperature: 870 to 940°C.
Operating conditions with range
Furnace Temperature : 870 – 940°C
Quenching Time : 30 – 90 minutes
Tempering Temperature : 150 - 250°C
Tempering Time : 80 - 120 minutes
Preheating : 0-150°C.

From the experimental result, the average effects of process variables under consideration on the obtainable surface hardness have been calculated and the same are presented in Table **5**.

The sample calculation for Average effect of Process variables on surface hardness is given below.

Variable: Furnace temperature, Variable level – Level 1, Material: EN 29 (Table **3**).

Average effect = (77+77.5+78.5+79.5+80.5+77+79.5+78.5+77.5)/9 = 78.39 HRA

Response graphs shown in Fig. (**4a-e**) are drawn using the values in Table **5**.

3.1.1. Influence of Process Variables on Hardness

ANOVA analysis is carried out to determine the influence of main variables on surface hardness and also to determine the percentage contributions of each variable. Table **6** shows the results of percentage contribution of each variable.

3.1.1.1. Model Calculation for EN 29

Correction factor, C.F $= [\sum yi]^2$ / Number of Experiments

$= [77+77.5+......79]^2/27 = 168823.14$

Total sum of squares, SST $= \sum yi^2 - \text{C.F} = 168866 - 168823.14 = 42.85$

Sum of Squares of Variables,

Variable A, SSA $= [\sum y1^2/9 + \sum y2^2/9 + \sum y3^2/9] - \text{C.F}$

$= [55303.36+56406.25+57121] - \text{C.F}$

$= 168830.61 - 168823.14$

$= 7.47$

Percentage contribution of each variable, A = (SSA/SST)*100

Table 3. Orthogonal Array for Gas Carburizing with Test Results and S/N Ratio Material: EN 29 (Surface Hardness Optimization)

S. No.	A	B	C	D	E	Hardness in HRA		Average HRA Value	S/N for HRA
						Trial 1	Trial 2		
1	870	60	150	80	NO	77	77	77	43.750
2	870	60	200	100	150	77	78	77.5	43.806
3	870	60	250	120	NO	77	80	78.5	43.919
4	870	90	150	100	150	78	81	79.5	44.029
5	870	90	200	120	NO	79	82	80.5	44.138
6	870	90	250	80	NO	77	77	77	43.750
7	870	120	150	120	NO	79	80	79.5	44.028
8	870	120	200	80	NO	78	79	78.5	43.918
9	870	120	250	100	150	77	78	77.5	43.806
10	910	60	150	80	150	79	79	79	43.973
11	910	60	200	100	NO	80	78	79	43.973
12	910	60	250	120	NO	81	80	80.5	44.136
13	910	90	150	100	NO	81	80	80.5	44.136
14	910	90	200	120	NO	81	80	80.5	44.136
15	910	90	250	80	150	80	79	79.5	44.028
16	910	120	150	120	NO	79	78	78.5	43.918
17	910	120	200	80	150	78	78	78	43.862
18	910	120	250	100	NO	77	77	77	43.750
19	940	60	150	80	NO	79	78	78.5	43.918
20	940	60	200	100	NO	79	77	78	43.863
21	940	60	250	120	150	81	78	79.5	44.029
22	940	90	150	100	NO	82	80	81	44.190
23	940	90	200	120	150	82	81	81.5	44.243
24	940	90	250	80	NO	81	78	79.5	44.029
25	940	120	150	120	150	80	79	79.5	44.028
26	940	120	200	80	NO	82	79	80.5	44.138
27	940	120	250	100	NO	80	78	79	43.973

= (7.47 /42.85) *100 = 17.43%

In the same way the percentage contribution of other variables are calculated.

Total contribution of variables,
(A+B+C+D+E+AxB+AxC)

= (17.43+18.21+4.34+7.94+10.19+25.43+3.98)

= 87.52%

∴ Error =12.48%

Optimum set of variables for surface hardness are found by adopting the higher is better strategy. The results are given in Table **7**.

3.2. Prediction of Mean Response – Surface Hardness

From Taguchis' methodology, equation (1) can be used to predict the surface hardness obtainable.

$$\beta = T + (HA_{opt} - T) + (HB_{opt} - T) + (HC_{opt} - T) + (HD_{opt} - T) + (HE_{opt} - T) \quad (1)$$

where,

β -predicted mean response

T-mean of all observed hardness values;

HA_{opt}, HB_{opt}, HCopt HD_{opt} and HE_{opt} – Hardness values obtained at optimum process variable condition.

3.2.1. Model Calculation for EN 29 Material

T= {(77+77.5+78.5+79.5+80.5+77+79.5+78.5+77.5+79+80.5+80.5+80.5+79.5+78.5+78+77+78.5+78+79.5+81+79.5+79.5+80.5+79)}/27 (from Table **3**)

T = 79.0741

$$\left.\begin{aligned} HA_{opt} &= 79.66667 \\ HB_{opt} &= 79.944 \\ HC_{opt} &= 79.3333 \\ HD_{opt} &= 79.830 \\ HE_{opt} &= 79.056 \end{aligned}\right\} \quad \text{From Table } \mathbf{5}$$

β (Surface hardness) = 79.0741+ (79.66667 –79.0741) + (79.944 –79.0741) + (79.3333 - 79.0741) + (79.830 - 79.0741) + (79.056 - 79.0741) = 81.5336 HRA

Similarly, for EN 34 the predicted mean response = 80.7802HRA optimum surface hardness value, for EN 29 is 81.5336HRA and for EN 34 is 80.7802HRA.

Table 4. Orthogonal Array for Gas Carburizing with Test Results and S/N Ratio Material: EN 34 (Surface Hardness Optimization)

S. No.	A	B	C	D	E	Hardness in HRA		Average HRA Value	S/N for HRA
						Trial 1	Trial 2		
1	870	60	150	80	NO	76	76	76	43.63687
2	870	60	200	100	150	79	79	79	43.97314
3	870	60	250	120	NO	80	78	79	43.97384
4	870	90	150	100	150	79	78	78.5	43.91817
5	870	90	200	120	NO	79	78	78.5	43.91817
6	870	90	250	80	NO	78	77	77.5	43.80681
7	870	120	150	120	NO	77	80	78.5	43.91958
8	870	120	200	80	NO	76	81	78.5	43.9224
9	870	120	250	100	150	79	79	79	43.97314
10	910	60	150	80	150	78	78	78	43.86249
11	910	60	200	100	NO	77	76	76.5	43.69401
12	910	60	250	120	NO	80	78	79	43.97384
13	910	90	150	100	NO	81	77	80	43.97592
14	910	90	200	120	NO	80	79	80	44.02811
15	910	90	250	80	150	78	80	80	43.97384
16	910	120	150	120	NO	78	82	80	44.08511
17	910	120	200	80	150	77	80	78.5	43.91958
18	910	120	250	100	NO	76	79	77.5	43.80826
19	940	60	150	80	NO	78	80	79	43.97384
20	940	60	200	100	NO	79	80	80	44.02811
21	940	60	250	120	150	80	79	80	44.02811
22	940	90	150	100	NO	81	79	80	44.08308
23	940	90	200	120	150	82	79	80.5	44.13803
24	940	90	250	80	NO	80	78	79	43.97384
25	940	120	150	120	150	79	79	79	43.97314
26	940	120	200	80	NO	78	78	78	43.86249
27	940	120	250	100	NO	77	81	79	43.97592

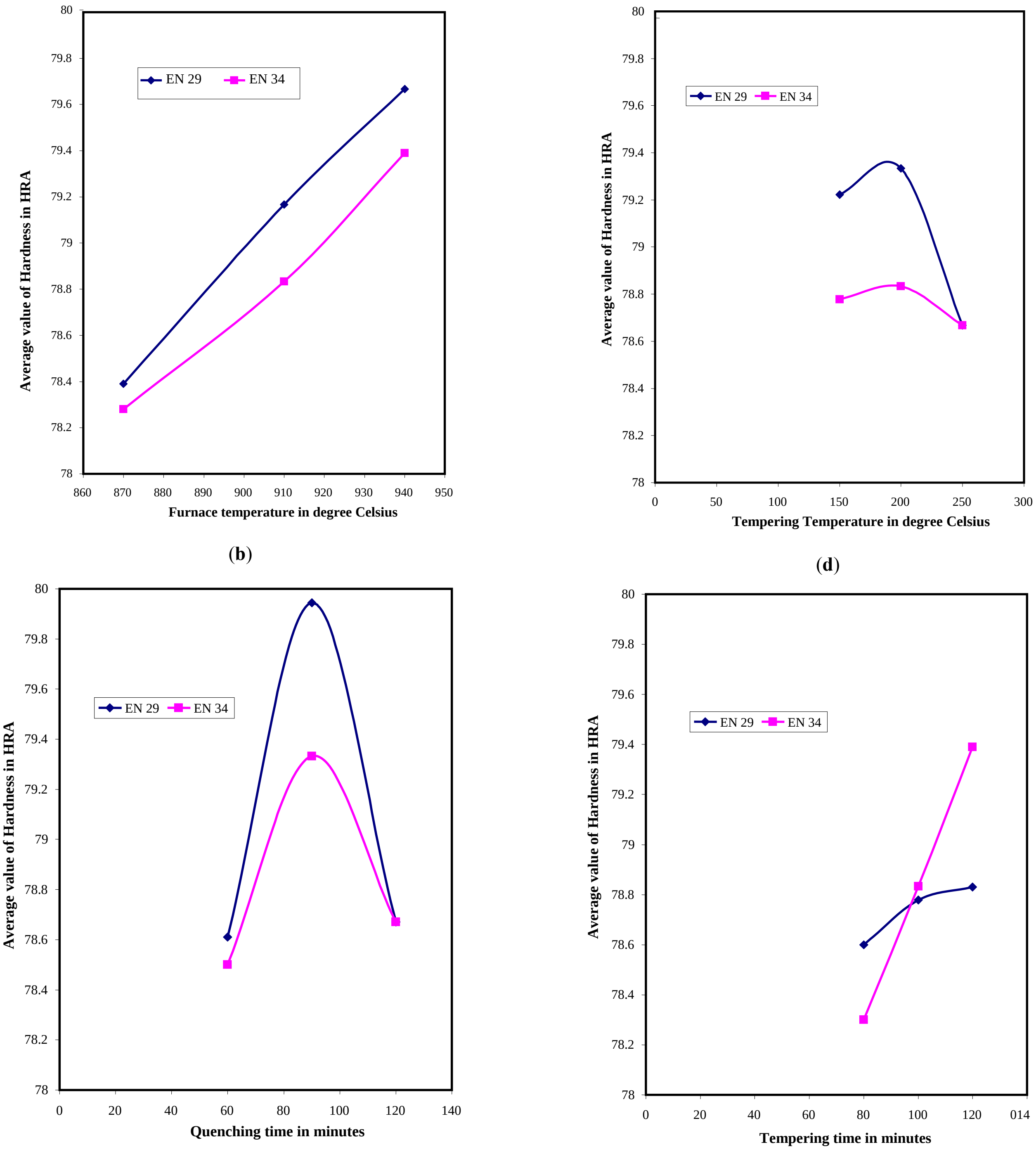
Table 5. Average Effect of Process Variables on Surface Hardness

Variables	Level 1		Level2		Level3	
	EN 29	EN 34	EN 29	EN 34	EN 29	EN 34
Furnace temperature	78.39	78.28	79.1667	78.8333	**79.66667**	**79.38889**
Quenching Time	78.61	78.5	**79.944**	**79.333**	78.67	78.67
Tempering Temperature	79.222	78.778	**79.3333**	**78.8333**	78.667	78.667
Tempering Time	78.6	78.3	78.778	78.833	**79.83**	**79.39**
Preheating	78.89	78.89	**79.056**	**79.167**	-	-

(a)

(c)

(Fig. 4) contd…..



(b)

(d)

(Fig. 4) contd…..

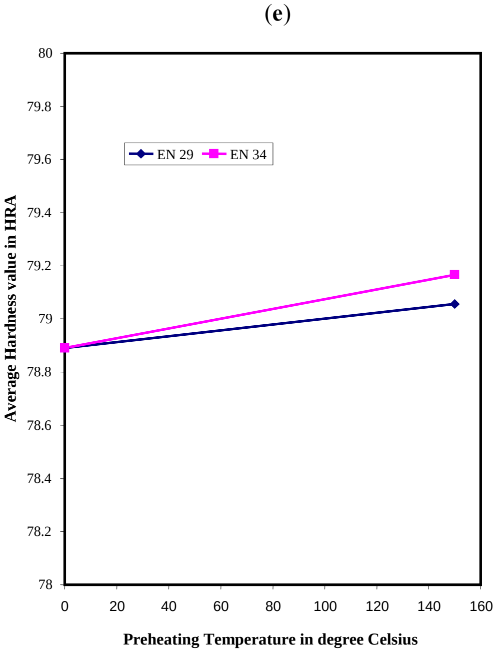


Fig. (4). (**a-e**) Process variables *vs* Hardness.

3.2. Hardness Optimization – Signal to Noise Ratio Method

Signal noise (S/N) ratio analysis estimates the effect of noise factors on the performance characteristics. It was developed as a proactive equivalent to the reactive loss function. Signal factors (**$\check{Y}$**) are set by the designer to obtain the intended value of the response variable. Noise factors (S^2) are not controlled or are very expensive or difficult to control (Harisingh and Pradeep Kumar 2004, Davilkar, *et al.*, 2003) [9]. The Gas carburizing conditions adopted in the experimentation are given in Table **1**, and the test results with S/N ratios are reported in Tables **3** and **4**.

S/N ratio for maximizing the response factor as the objective (Maximizing the surface hardness) is determined from the equation (2).

$$S/N = -10 \text{ Log}_{10} [1/\Sigma yi^2 * n] \qquad (2)$$

where, yi - the experimental response values for the trials, and n - number of trials.

Optimum condition for surface hardness are found by adopting the higher the S/N ratio is better as the strategy and results are given in the Table **7**. The optimum condition result obtained in S/N method matches with the optimum result obtained from the response graph analysis.

3.2.1. Model Calculation for the Material EN 29

S/N ratio for maximizing the Case depth (10th Experiment run)

$$S/N = -\ 10 \log_{10} \{1/\ (79^2 + 79^2\ )*2\} = 43.97314$$

4. CASE-DEPTH IN PINION

Depth of hardness penetration in any surface heat treated material is an important factor in deciding the reliability and life of the part [10, 11]. Hence, it is necessary to measure the

Table 6. Percentage Contribution of Each Variable on Surface Hardness

Variables	Surface Hardness	
	EN 29	**EN 34**
Furnace temperature	**17.43%**	**17.79%**
Quenching Time	**18.21%**	**15.29%**
Tempering Temperature	**4.34%**	**6.34%**
Tempering Time	**7.94%**	**8.25%**
Preheating	**10.19%**	**12.36%**
Furnace Temperature and Quenching time	**25.43%**	**24.91%**
Furnace Temperature and Tempering Temperature	**3.98%**	**5.28%**
Error	**12.48%**	**9.78%**

Table 7. Optimum Conditions for Surface Hardness

Variables	Surface Hardness	
	EN 29	**EN 34**
Furnace temperature	**940°C**	**940°C**
Quenching Time	**90 minutes**	**90 minutes**
Tempering Temperature	**200°C**	**200°C**
Tempering Time	**120 minutes**	**120 minutes**
Preheating	**150°C**	**150°C**

case depth of the hardened layer in the given part. Case depth is defined as "the perpendicular distance from the surface of the metallic material to the point at which the change in hardness, chemical composition or microstructure of the case and core cannot be distinguished" (Rajan, T.V., *et al*., 1998) [12].

Process variable optimization to obtain the required case depth in the Pinion material (Fig. **5**) subjected for Gas carburizing is done both by Response graph analysis and by Signal to Noise ratio analysis. The trials are conducted as per Taguchi's L27 orthogonal array approach.

4.1. Case Depth Optimization – Response Graph Method

The conditions underwhich the Gas carburizing experiments have been conducted to arrive at the optimum process variables are given in Table **1**. The test results are reported in Tables **8** and **9**.

Fig. (5). Specimen of pinion material for case depth analysis.

Table 8. Orthogonal Array of Gas Carburizing with Test Results and S/N Ratio Material: EN 29 (Case -Depth Optimization)

S. No.	A	B	C	D	E	Case Depth in mm		Average Case Depth Value	S/N for Case Depth
						Trial 1	Trial 2		
1	870	60	150	80	NO	0.7	0.7	0.7	2.92
2	870	60	200	100	150	0.9	1	0.95	5.59
3	870	60	250	120	NO	0.7	0.8	0.75	3.54
4	870	90	150	100	150	0.8	0.9	0.85	4.62
5	870	90	200	120	NO	0.9	0.9	0.9	5.11
6	870	90	250	80	NO	1.0	0.8	0.9	5.16
7	870	120	150	120	NO	0.8	0.9	0.85	4.62
8	870	120	200	80	NO	0.7	0.8	0.75	3.54
9	870	120	250	100	150	0.8	0.8	0.8	4.08
10	910	60	150	80	150	0.9	1.0	0.95	5.59
11	910	60	200	100	NO	1.0	0.9	0.95	5.59
12	910	60	250	120	NO	0.8	0.8	0.8	4.08
13	910	90	150	100	NO	0.7	0.7	0.7	2.92
14	910	90	200	120	NO	0.6	0.6	0.6	1.58
15	910	90	250	80	150	0.8	0.8	0.8	4.08
16	910	120	150	120	NO	0.7	0.8	0.75	3.54
17	910	120	200	80	150	0.7	0.7	0.7	2.92
18	910	120	250	100	NO	0.8	0.8	0.8	4.08
19	940	60	150	80	NO	0.9	0.8	0.85	4.62
20	940	60	200	100	NO	0.8	0.7	0.75	3.54
21	940	60	250	120	150	0.7	0.6	0.65	2.3
22	940	90	150	100	NO	0.6	0.8	0.7	3.01
23	**940**	**90**	**200**	**120**	**150**	**1.0**	**1.0**	**1.0**	**6.02**
24	940	90	250	80	NO	0.9	0.8	0.85	4.62
25	940	120	150	120	150	0.9	1	0.95	5.59
26	940	120	200	80	NO	1	0.9	0.95	5.59
27	940	120	250	100	NO	0.8	0.8	0.8	4.08

Table 9. Orthogonal Array of Gas Carburizing with Test Results and S/N Ratio Material: EN 34 (Case-Depth Optimization)

S.No.	A	B	C	D	E	Case Depth in mm		Average Case Depth Value	S/N for Case Depth
						Trial 1	Trial 2		
1	870	60	150	80	NO	0.8	0.6	0.7	3.01
2	870	60	200	100	150	0.7	0.7	0.7	2.92
3	870	60	250	120	NO	0.7	0.7	0.7	2.92
4	870	90	150	100	150	0.8	0.9	0.85	4.62
5	870	90	200	120	NO	0.9	0.8	0.85	4.62
6	870	90	250	80	NO	0.6	0.7	0.65	2.3
7	870	120	150	120	NO	0.7	0.8	0.75	3.54
8	870	120	200	80	NO	0.8	0.9	0.85	4.62
9	870	120	250	100	150	0.8	0.9	0.85	4.62
10	910	60	150	80	150	0.8	0.8	0.8	4.08
11	910	60	200	100	NO	0.8	0.8	0.8	4.08
12	910	60	250	120	NO	0.8	0.8	0.8	4.08
13	910	90	150	100	NO	0.8	0.7	0.75	3.54
14	910	90	200	120	NO	0.9	0.8	0.85	4.62
15	910	90	250	80	150	0.9	0.8	0.85	4.62
16	910	120	150	120	NO	0.8	0.7	0.75	3.54
17	910	120	200	80	150	0.7	0.6	0.65	2.3
18	910	120	250	100	NO	0.7	0.8	0.75	3.54
19	940	60	150	80	NO	0.8	0.8	0.8	4.08
20	940	60	200	100	NO	0.8	0.8	0.8	4.08
21	940	60	250	120	150	0.9	0.8	0.85	4.62
22	940	90	150	100	NO	0.8	0.8	0.8	4.08
23	**940**	**90**	**200**	**120**	**150**	**1.0**	**0.90**	**0.95**	**5.59**
24	940	90	250	80	NO	0.9	0.8	0.85	4.62
25	940	120	150	120	150	0.7	0.8	0.75	3.54
26	940	120	200	80	NO	0.9	0.8	0.85	4.62
27	940	120	250	100	NO	0.6	0.6	0.6	1.58

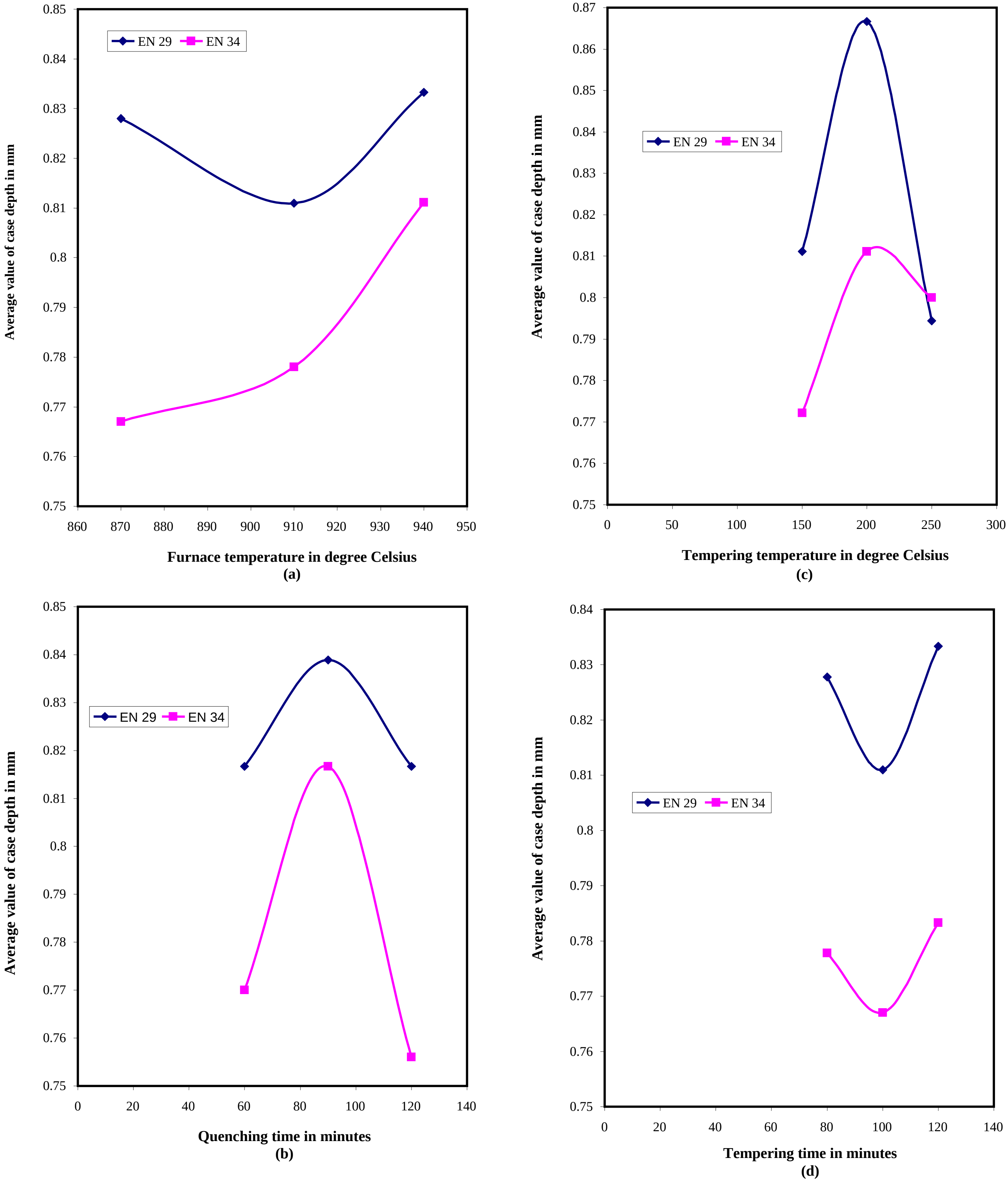
The average effects of main factors on case depth are given in Table **10** for the materials EN 29 and EN 34 respectively.

Response graphs are drawn using Table **10**. Fig. (**6a-e**) (Response graphs) shows the influence of process variables on the case depth for the Materials EN 29 and EN 34.

Table 10. Average Effect of Process Variables on Case Depth

Variables	Level 1		Level2		Level3	
	EN 29	EN 34	EN 29	EN 34	EN 29	EN 34
Furnace temperature	0.828	0.767	0.811	0.778	**0.8333**	**0.8111**
Quenching Time	0.81667	0.77	**0.83889**	**0.8167**	0.8167	0.756
Tempering Temperature	0.8111	0.7722	**0.86667**	**0.811111**	0.79444	0.8000
Tempering Time	0.8278	0.7778	0.811	0.767	**0.8333**	**0.783333**
Preheating	0.7778	0.7556	**0.85**	**0.8056**	-	-

(Fig. 6) contd……



(a) (c)

(b) (d)

(Fig. 6) contd……

(e)

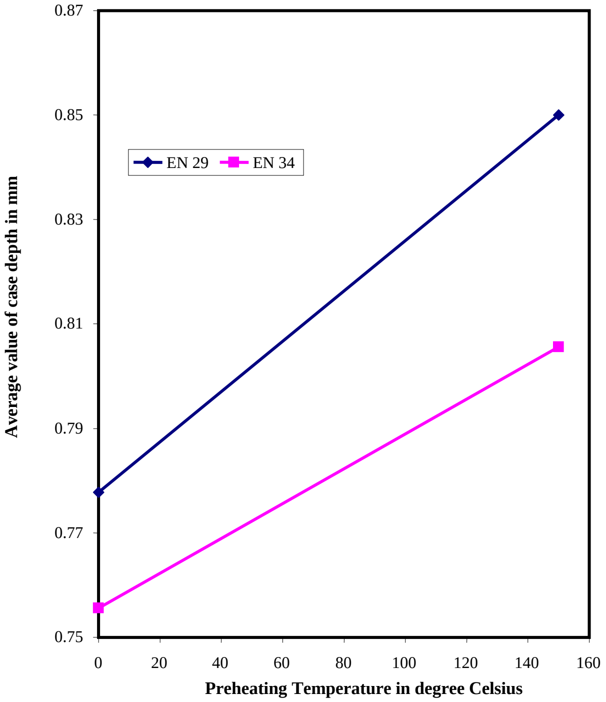


Fig. (6). (**a-e**) Process variables *vs* Case-depth.

4.1.1. Influence of Main Variables on Case Depth

ANOVA analysis are carried out to determine the influence of main variables on case depth and also to determine the percentage contributions of each variable. Table **11** shows the results of percentage contribution of each variable.

4.1.1.1. Model Calculation for EN 29

Correction factor, C.F = $[\sum yi]^2$ / Number of Experiments

$= [0.7+0.95+\ldots\ldots0.8]^2 /27 = 17.925$

Total sum of squares, SST = $\sum yi^2 - \text{C.F}$

$= 18.215-17.925 = 0.29$

Sum of Squares of variables,

Variable A, SSA $= [\sum y1^2/9 + \sum y2^2/9 + \sum y3^2/9] - \text{C.F}$

$= [6.166+5.522+6.25]-\text{C.F}$

$= 17.938-17.925$

$= 0.013$

Percentage contribution of

Variable, A = (SSA/SST)*100

$= (0.013/0.29)*100 = 4.48\%$

In the same way the percentage contribution of other variables are calculated.

Total contribution of variables

$= (A + B + C + D + E + AxB + AxC)$

$= (4.48+5.99+9.13+11.28+16.28+29.17+8.18)$

$= 84.51\%$

∴ Error $= 12.48\%$

Table 11. Percentage Contribution of Each Variable on Case Depth

Variables	Case Depth	
	EN 29	**EN 34**
Furnace temperature	**4.48%**	**5.43%**
Quenching Time	**5.99%**	**7.14%**
Tempering Temperature	**9.13%**	**11.41%**
Tempering Time	**11.28%**	**12.78%**
Preheating	**16.28%**	**15.49%**
Furnace Temperature and Quenching time	**29.17%**	**28.18%**
Furnace Temperature and Tempering temperature	**8.18%**	**9.16%**
Error	**12.48%**	**10.41%**

Optimum conditions for Case depth are found by adopting the higher is better strategy. The results are given in Table **7**. It is significant to note that the optimum conditions for hardness and case depth are the same.

4.2. Prediction of Mean Response – Case Depth

$$\beta = T + (CA_{opt} - T) + (CB_{opt} - T) + (CC_{opt} - T) + (CD_{opt} - T) + (CE_{opt} - T) \quad (3)$$

where,

β -predicted mean response

T-mean of all observed case depth values;

CAopt, CBopt, CCopt, CD $_{opt}$, and CE $_{opt}$ - Case depth values obtained at optimum process variable conditions

4.2.1. Model Calculation for EN29 Material

T= {(0.70+0.95+0.75+0.85+0.90+0.90+0.85+0.75+0.80+0.95+0.95+0.80+0.70+0.60+0.80+0.75+0.70+0.80+0.85+0.75+0.65+0.70+1.00+0.85+0.95+0.95+0.80)}/27 (from Table **8**)

$$\left.\begin{aligned} &T = 0.81481 \\ &CA_{opt} = 0.8333 \\ &CB_{opt} = 0.83889 \\ &CC_{opt} = 0.8667 \\ &CD_{opt} = 0.8333 \end{aligned}\right\} \text{from Table } \mathbf{11}$$

$CE_{opt} = 0.85$

β (Case depth) $= 0.81481 + (0.8333 - 0.81481) + (0.83889 - 0.81481) + (0.8667 - 0.81481) + (0.8333 - 0.81481) + (0.85 - 0.81481)$

$= 0.879605$ mm

Similarly for EN 34 the predicted mean response = 0.96265 mm.

Optimum Case depth value, for EN 29 = **0.96265 mm** and for EN 34 = **0.8945 mm**.

4.3. Case Depth Optimization – Signal to Noise Ratio Method

Gas carburizing conditions adopted in the experimentation are given in Table **1**, and the test results with S/N ratio are given in Tables **8** and **9**. Optimum condition for Case depth are found by adopting the higher the S/N ratio is better as the strategy and results are given in the Table **7**. The optimum condition result obtained in S/N method matches with the optimum result obtained from the response graph analysis.

4.3.1. Model Calculation for the Material EN 29

S/N ratio for maximizing the Case depth (13th Experiment run)

$$S/N = -10 \log_{10} \{1/(0.7^2 + 0.7^2)*2\} = 2.92256$$

5. INDUCTION HARDENING PROCESS VARIABLES OPTIMIZATION USING FACTORIAL METHOD

In this study 3^3 Factorial Design Matrix is used to optimize the process variables for obtaining improved surface integrity of surface hardened components [13]. The experiments are conducted to study the influence of process variables on Surface hardness and Case depth as per Classical DOE. All these trials have been carried out by Randomization method. ANOVA analysis with F-Test has been carried out to determine the influence of each factor and their interactions. Regression analysis is done to develop a modeling equation to predict the hardness [14]. AISI 4340 and AISI 1055 are the materials used in this Induction hardening process experiment. The normal procedure followed in converting the raw material into a finished product is shown in Fig. (**7**).

5.1. Hardness Optimization on Rack

Induction surface hardened low alloyed medium carbon steels are widely used for critical automotive and machine applications such as rack and pinion, propulsion shaft, crankshaft and steering knuckles, which require high surface hardness with low distortion. Rack is a critical component used in the power steering of automobiles. Normally, the rack is surface hardened by Induction hardening to withstand the wear loads. Literatures show that in the case of induction hardening process, the power potential, scan speed (Heating Time) [15]. Quench flow rate and frequency are the major influential variables, which controls the surface hardness, hardness penetration depth (HPD) and level of distortion. The present study demonstrates the optimization of critical process variables involved in the Induction hardening of a Rack material used in the power steering of the automobile [16].

In order to study the influence of process variables on the hardness in the AISI 4340 and AISI 1055 Rack materials, Induction hardening experiments are conducted. Experimental investigations are carried out in Electro magnetic Induction hardening Furnace (Fig. **8**). Table **12** shows the details about the operating conditions. The specifications of induction hardening Furnace are given in Table **13**. Tables **14** and **15** shows the Experimental results in the 3^3 Design Matrix for the materials AISI 4340 and AISI 1055 respectively. Tables **16** and **17** show the ANOVA with F-Test of the materials AISI 4340 and AISI 1055 respectively.

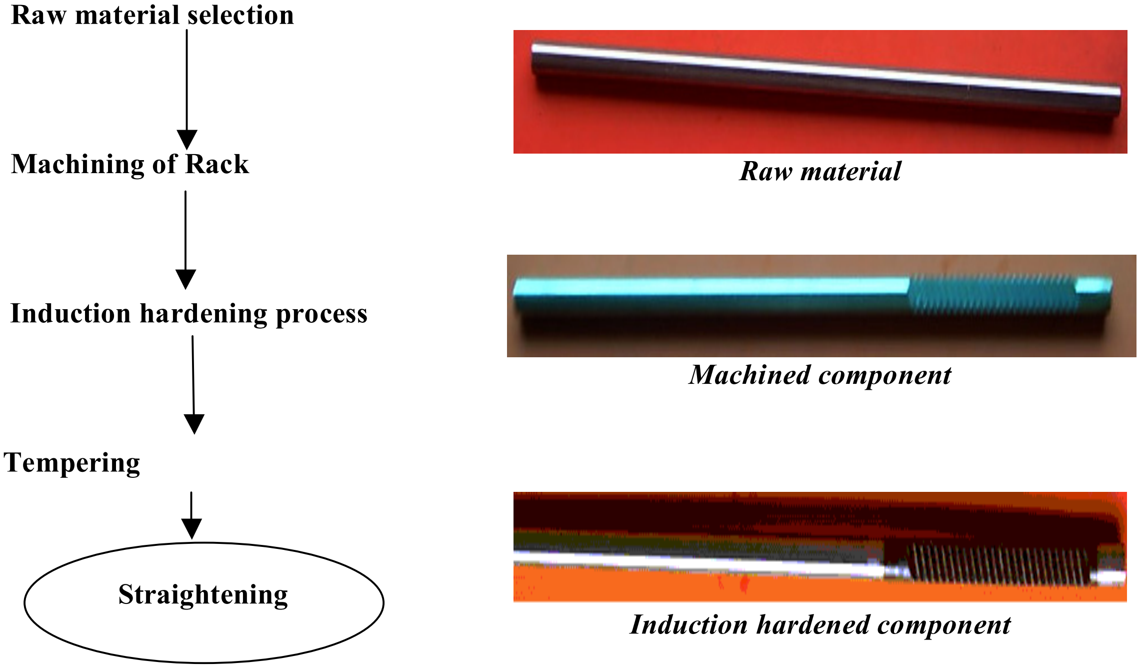


Fig. (7). Sequence of operation in induction hardening.

Table 12. Induction Hardening Operating Conditions

S. No.	Variables	Unit	Notation	Levels actual			Code		
				Low	Medium	High	Low	Medium	High
1	Power potential	kW/inch^2	P	5.5	7.05	8.5	L1	L2	L3
2	Scan speed	m/min	S	1.34	1.72	2.14	L1	L2	L3
3	Quench flow rate	Litres/min	Q	15	17.5	20	L1	L2	L3

The experiments have been conducted based on 3^3 full factorial DOE.

Fig. (8). Induction hardening furnace.

Table 13. Specifications and Operating Conditions of Induction Hardening

Material Used : AISI – Medium Carbon steel materials
Diameter : 23 mm ; Heating length : 200 mm
Furnace Details:
440V, 3mm coupling distance Inducto Heat induction hardening device
Frequency ; 1000 to 10,000 cycles per second
Temperature: 750 to 800ºC.
Operating conditions with range
Power Potential - 5.5 – 8.5 kW/inch^2
Scan speed - 1.72 -2.5 m/minutes
Quench Flow rate - 15 - 20 Litres/minutes

5.1.1. Influence of Main Variables on Surface Hardness of Rack Material

5.1.1.1. Model Calculation (AISI 4340)

Total sum of the run = (83+81+82+….63+66+63) = 5958

Number of Treatments = 3 (3 Factors)

Number of Levels = 3

Number of replicates (r) = 3

Total of the observations under all factor levels = N = abcr = 3x3x3x3=81

Correction factor, (C) = $(5958)^2/81$

= 438244

Sum of Squares of Treatment, (SST) = $(83^2+81^2+82^2+\ldots\ 63^2+66^2+63^2)$

= (440832) -C

= 2588

Sum of Squares of Treatment with replicates, (SST_r) = $1/3(246^2+249^2+\ldots 192^2)$

= 1/3(1322244)-C

= 2504

Sum of Squares of Replicate, (SSR)

= $1/27(1980^2+1989^2+1989^2)$-C

= 2

Sum of Squares of Error, (SSE) = SST-SST_r – SSR

= 2588-2504-2

= 82

	S				Q				Q		
2154	729	711	714	P				2034	690	984	660
1974	654	660	660					1956	648	654	654
1830	651	585	594					1968	669	654	645
5958	2034	1956	1968	2154	723	723	708	**5958**	2007	1992	1959
P				1974	657	660	657	S			
				1830	627	609	594				
				5958	2007	1992	1959				

Table 14. 3^3 Design Matrix for Induction Hardening with Test Results Material: AISI 4340 (Surface Hardness)

S. No.	P	S	Q	Hardness in HRA		
				Trial 1	Trial 2	Trial 3
1	5.5	1.34	15	83	81	82
2	5.5	1.34	17.5	84	83	82
3	5.5	1.34	20	76	78	80
4	5.5	1.72	15	79	80	78
5	5.5	1.72	17.5	79	81	80
6	5.5	1.72	20	76	80	78
7	5.5	2.14	15	79	81	80
8	5.5	2.14	17.5	79	77	78
9	5.5	2.14	20	80	81	79
10	7.05	1.34	15	74	72	76
11	7.05	1.34	17.5	69	71	73
12	7.05	1.34	20	73	72	74
13	7.05	1.72	15	69	71	70
14	7.05	1.72	17.5	75	75	75
15	7.05	1.72	20	76	74	75
16	7.05	2.14	15	76	74	75
17	7.05	2.14	17.5	74	73	75
18	7.05	2.14	20	70	72	71
19	8.5	1.34	15	75	73	74
20	8.5	1.34	17.5	74	75	73
21	8.5	1.34	20	68	70	69
22	8.5	1.72	15	68	66	67
23	8.5	1.72	17.5	64	63	62
24	8.5	1.72	20	65	65	65
25	8.5	2.14	15	66	68	70
26	8.5	2.14	17.5	66	67	65
27	8.5	2.14	20	63	66	63

5.1.2. Sum of Squares of Main Effect (P, S and Q)

Sum of Square of Power Potential, SSP = $[1/27(2154^2+1974^2+1830^2)]$-C = 1952

Sum of Square of Scan Speed, SSS = $[1/27(2034^2+1956^2+1968^2)]$-C = 130.66

Sum of Square of Quench flow rate, SSQ = $[1/27(2007^2+1992^2+1959^2)]$-C= 44.66

5.1.3. Two-Way Interactions of Sum of Squares (PS, SQ and PQ)

Sum of Square of Power Potential and Scan Speed

= $[1/9(729^2+\ldots 594^2)]$ –C=177.34

Sum of Square of Scan Speed and Quench flow rate

= $[1/9(690^2+\ldots.645^2)]$-C=46.68

Sum of Square of Power Potential and Quench flow rate

= $[1/9(723^2+\ldots 594^2)]$-C= 33.34

5.1.4. Three Way Interactions of Sum of Square (PSQ)

Sum of Square of Power Potential, Scan speed and

Quench flow rate

= SST_r - {SSP-SSS-SSQ-SSPS-SSSQ-SSPQ}

= 2504-1952-130.66-44.66-177.34-46.68-33.34 = 119.32

Table 15. 3^3 **Design Matrix for Induction Hardening with Test results Material: AISI 1055 (Surface Hardness)**

S. No.	P	S	Q	Hardness in HRA		
				Trial 1	Trial 2	Trial 3
1	5.5	1.34	15	83	84	85
2	5.5	1.34	17.5	82	83	84
3	5.5	1.34	20	83	84	85
4	5.5	1.72	15	78	81	81
5	5.5	1.72	17.5	79	80	81
6	5.5	1.72	20	83	84	85
7	5.5	2.14	15	80	81	82
8	5.5	2.14	17.5	82	83	84
9	5.5	2.14	20	81	82	83
10	7.05	1.34	15	77	78	79
11	7.05	1.34	17.5	77	81	82
12	7.05	1.34	20	77	79	78
13	7.05	1.72	15	69	71	70
14	7.05	1.72	17.5	78	76	77
15	7.05	1.72	20	79	81	80
16	7.05	2.14	15	75	76	74
17	7.05	2.14	17.5	74	72	73
18	7.05	2.14	20	73	74	72
19	8.5	1.34	15	78	77	79
20	8.5	1.34	17.5	71	69	70
21	8.5	1.34	20	69	67	68
22	8.5	1.72	15	69	71	70
23	8.5	1.72	17.5	75	74	73
24	8.5	1.72	20	71	70	69
25	8.5	2.14	15	68	66	67
26	8.5	2.14	17.5	70	69	68
27	8.5	2.14	20	72	71	73

Table 16. ANOVA with F-Test Material: AISI 4340 (Surface Hardness)

Variable	Sum of Squares	Degrees of Freedom	Mean Square	F	Significant Ranking
Replicates	2	2	1	0.6345	-
P	1952	2	976	619.289	1
S	130.66	2	65.33	41.453	2
Q	44.66	2	22.33	14.168	4
PS	177.34	4	44.335	28.131	3
SQ	46.68	4	11.67	7.404	6
PQ	33.34	4	8.335	5.288	7
PSQ	119.32	8	14.915	9.463	5
ERROR	82	52	1.576	-	
TOTAL	-	80			

Regression analysis is done using MATLAB and the Regression equations (Equation to predict the hardness of the material AISI 4340) are found and given below.

AISI 4340

```
Coeff =
1.0000 5.5000 1.3400 15.0000     82
1.0000 5.5000 1.3400 17.5000     83
1.0000 5.5000 1.3400 20.0000     78
1.0000 5.5000 1.7200 15.0000     78
1.0000 5.5000 1.7200 17.5000     80
1.0000 5.5000 1.7200 20.0000     79
1.0000 5.5000 2.1400 15.0000     80
1.0000 5.5000 2.1400 17.5000     77
1.0000 5.5000 2.1400 20.0000     80
1.0000 7.0500 1.3400 15.0000     74
1.0000 7.0500 1.3400 17.5000     72
1.0000 7.0500 1.3400 20.0000     73
1.0000 7.0500 1.7200 15.0000     70
1.0000 7.0500 1.7200 17.5000     75
1.0000 7.0500 1.7200 20.0000     75
1.0000 7.0500 2.1400 15.0000     74
1.0000 7.0500 2.1400 17.5000     72
1.0000 7.0500 2.1400 20.0000     75
1.0000 8.5000 1.3400 15.0000     75
1.0000 8.5000 1.3400 17.5000     69
1.0000 8.5000 1.3400 20.0000     67
1.0000 8.5000 1.7200 15.0000     68
1.0000 8.5000 1.7200 17.5000     63
1.0000 8.5000 1.7200 20.0000     65
1.0000 8.5000 2.1400 15.0000     68
1.0000 8.5000 2.1400 17.5000     66
1.0000 8.5000 2.1400 20.0000     64
```

The coefficients for the formation of hardness equation are,

111.5611
-4.1474
-2.3059
-0.2889

Equation to Predict the Hardness of the Material AISI 4340

$$Y_H = 111.5611-4.1474P-2.3059S-0.2889Q \qquad (4)$$

Similarly, for the material AISI 1055, equation to predict the hardness is given by

$$Y_H = 106.7885-3.8179P-3.38671S-0.1778Q \qquad (5)$$

5.2. Case Depth Optimization in Rack

After Induction hardening a steel component is usually hardness/ Case depth tested. And the value obtained is a good indication of the effectiveness of the treatment [17]. The case depth can be measured either by Visual examination or by Hardness measurement. The case depth/Hardness test is carried out by pressing a ball or point with a predetermined force into the surface of the specimen. The hardness figure is function of the size of the indentation for the Brinell (HB) and Vickers (HV), tests and of the depth of the penetration for Rockwell (HRC) test. The above three methods are the most commonly used tests and each has its special range of application and between them they cover almost the whole for the hardness/Case depth field that is of interest of the steel producer and user [18-19].

The present study explains the optimization [20] of critical process variables involved in the Induction hardening of a Rack material (Fig. **10**) used in the power steering of the automobile to get higher case depth [21].

In order to study the influence of process variables on the Case depth of the Rack material below the teeth and back of the bar for the AISI 4340 and AISI 1055 Rack materials Induction hardening experiments are conducted.

Table **12** shows the details about the operating conditions.

Table 17. ANOVA with F-Test Material: AISI 1055 (Surface Hardness)

Variable	Sum of Squares	Degrees of Freedom	Mean Square	F	Significant Ranking
Replicates	6.745	2	3.3725	2.771	-
P	1774.89	2	887.445	729.207	1
S	134.22	2	67.11	55.143	2
Q	11.56	2	5.78	4.749	7
PS	40.45	4	10.1125	8.309	6
SQ	157.78	4	39.445	32.411	3
PQ	53.11	4	13.2775	10.910	5
PSQ	229.55	8	28.69	23.574	4
ERROR	63.295	52	1.217	-	
TOTAL	-	80			

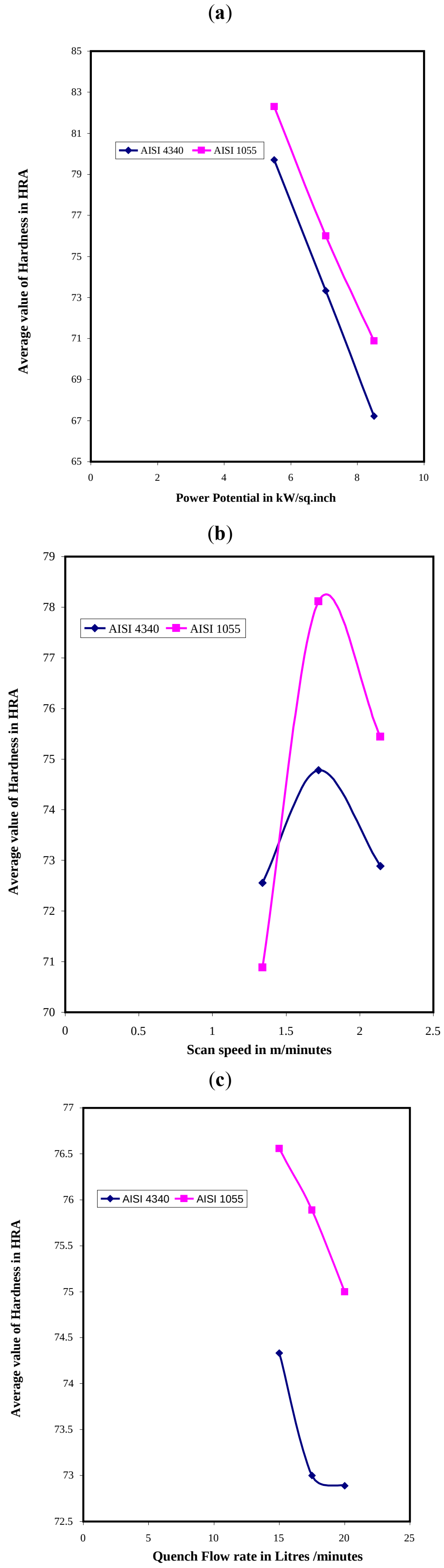


Fig. (9). (a-c) Process variables *vs* surface hardness.

Fig. (10). Cut section of an induction hardened rack component.

Tables **18** and **19** shows the Experimental results in the 3^3 Design Matrix for the materials AISI 4340 and AISI 1055 respectively.

Tables **20** and **21** shows the ANOVA with F-Test of the material AISI 4340 for case depth Below the teeth and Back of the bar respectively.

Tables **22** and **23** shows the ANOVA with F-Test of the material AISI 1055 for case depth Below the teeth and Back of the bar respectively.

5.2.1. Influence of Main Variables on Case Depth of Rack Material (Below the Teeth and Back of the Bar for AISI 4340)

5.2.1.1. Model Calculation (Case Depth – Below the Teeth)

Total sum of the run

= (2.4+2.6+2.5+….1.6+1.4+1.5) = 142

Number of Treatments

= 3 (3 Factors)

Number of Levels = 3

Number of replicates
(r) = 3

Total of the observations

under all factor levels

= N = abcr = 3x3x3x3=81

Correction factor, (C)

= $(142)^2/81$

= 248.93

Sum of Squares of Treatment, (SST)

= $(2.4^2 + 2.6^2 + 2.5^2 + \ldots 1.6^2 + 1.4^2 + 1.5^2)$

= (261.56)-C

= 12.63

Sum of Squares of Treatment with replicates, (SST_r)

= $1/3(7.5^2 + 6.9^2 + \ldots 4.5^2)$

= 1/3(783.4)-C

= 12.20

Sum of Squares of Replicate, (SSR)

= $1/27(47.3^2 + 47.1^2 + 47.6^2)$-C

= 0.013

Sum of Squares of Error, (SSE)

= $SST - SST_r - SSR$

= 12.63-12.20-0.013

= 0.417

	S				Q				Q		
57.6	20.1	20.1	17.4	P				49.3	17.1	16.6	15.6
46.2	16.2	14.4	15.6					46.8	14.4	16.8	15.6
38.2	13.0	12.3	12.9					45.9	17.4	14.7	13.8
142	49.3	46.8	45.9	57.6	21	18.6	18	**142**	48.9	48.1	45
P				46.2	15.6	15.9	14.7	S			
				38.2	12.3	13.6	12.3				
				142	48.9	48.1	45.0				

Table 18. 3^3 Design Matrix for Induction Hardening with Test Results Material: AISI 4340 (Case Depth)

S. No.	P	S	Q	Case Depth Below the Teeth			Case Depth Back of the Bar		
				Trial 1	Trial 2	Trial 3	Trial 1	Trial 2	Trial 3
1	5.5	1.34	15	2.4	2.6	2.5	4.3	4.3	4.3
2	5.5	1.34	17.5	2.2	2.4	2.3	4.1	4.0	4.2
3	5.5	1.34	20	2	1.8	1.9	3.2	3.4	3.3
4	5.5	1.72	15	2.4	2.4	2.4	2.9	2.7	2.8
5	5.5	1.72	17.5	2	2	2	1.2	1.0	1.1
6	5.5	1.72	20	2.4	2.2	2.3	2.4	2.2	2.3
7	5.5	2.14	15	2.5	2.5	2.5	4.0	4.0	4.0
8	5.5	2.14	17.5	1.4	1.5	1.6	2.8	3.0	2.9
9	5.5	2.14	20	2	1.7	1.7	3.4	3.6	3.2
10	7.05	1.34	15	1.8	1.9	2	2.7	2.7	2.7
11	7.05	1.34	17.5	1.5	1.5	1.5	2.6	2.7	2.8
12	7.05	1.34	20	2	2	2	1.8	2.2	2.0
13	7.05	1.72	15	1.5	1.5	1.5	3	3.1	3.2
14	7.05	1.72	17.5	1.6	1.7	1.8	2.3	2.2	2.4
15	7.05	1.72	20	1.5	1.8	1.5	3.4	3.5	3.6
16	7.05	2.14	15	1.9	1.7	1.8	2.7	2.5	2.9
17	7.05	2.14	17.5	2.1	2	2.2	2.9	2.9	2.9
18	7.05	2.14	20	1.3	1.3	1.3	1.7	1.7	2.0
19	8.5	1.34	15	1.3	1.3	1.3	1.9	1.8	2.7
20	8.5	1.34	17.5	1.6	1.8	1.8	1.5	1.7	1.6
21	8.5	1.34	20	1.2	1.3	1.4	1.2	1.2	1.2
22	8.5	1.72	15	1.3	1.3	1.3	1.3	1.4	1.5
23	8.5	1.72	17.5	1.6	1.5	1.4	1.0	0.8	0.9
24	8.5	1.72	20	1.3	1.2	1.4	1.3	1.3	1.0
25	8.5	2.14	15	1.5	1.5	1.5	1.2	1.2	1.2
26	8.5	2.14	17.5	1.4	1.3	1.2	1.2	1.3	1.4
27	8.5	2.14	20	1.6	1.4	1.5	1.0	1.1	1.2

Table 19. 3^3 **Design Matrix for Induction Hardening with Test Results Material: AISI 1055 (Case Depth)**

S. No.	P	S	Q	Case Depth Below the Teeth			Case Depth Back of the Bar		
				Trial 1	Trial 2	Trial 3	Trial 1	Trial 2	Trial 3
1	5.5	1.34	15	2.2	2.4	2.3	4	4.1	4.2
2	5.5	1.34	17.5	2.0	2.2	2.1	3.9	4	3.8
3	5.5	1.34	20	2.2	2.0	2.1	4.1	4.2	4
4	5.5	1.72	15	1.9	1.9	1.9	3.7	3.6	3.8
5	5.5	1.72	17.5	1.9	2.0	1.8	3.2	3	3.1
6	5.5	1.72	20	2.1	2.0	2.2	3.5	3.4	3.3
7	5.5	2.14	15	2.3	2.1	2.2	3	3	3
8	5.5	2.14	17.5	1.1	1.1	1.1	2.9	2.7	2.8
9	5.5	2.14	20	1.8	1.6	1.7	4	4	4
10	7.05	1.34	15	1.8	1.8	1.8	3.7	3.9	3.8
11	7.05	1.34	17.5	1.8	1.8	1.8	3.3	3.1	3.2
12	7.05	1.34	20	1.7	1.6	1.8	3.4	3.3	3.5
13	7.05	1.72	15	1.6	1.6	1.6	3.5	3.5	3.5
14	7.05	1.72	17.5	2.0	1.9	1.8	3.7	3.6	3.5
15	7.05	1.72	20	1.7	2.0	1.7	2	2.2	2.1
16	7.05	2.14	15	1.7	1.7	2.0	2.8	2.7	2.9
17	7.05	2.14	17.5	1.3	1.1	1.2	3.2	2.8	3
18	7.05	2.14	20	1.8	1.9	1.7	2.1	2	2.2
19	8.5	1.34	15	1.2	1.4	1.3	2	1.7	1.7
20	8.5	1.34	17.5	1.4	1.4	1.4	1.3	1.2	1.1
21	8.5	1.34	20	1.3	1.2	1.4	1.7	1.5	1.6
22	8.5	1.72	15	1.4	1.6	1.8	1.9	1.8	1.7
23	8.5	1.72	17.5	1.5	1.7	1.9	0.8	1	0.9
24	8.5	1.72	20	1.4	1.3	1.5	1.3	1.2	1.1
25	8.5	2.14	15	1.3	1.4	1.2	1.6	1.6	1.6
26	8.5	2.14	17.5	1.4	1.6	1.5	0.8	1	0.9
27	8.5	2.14	20	1.3	1.5	1.4	1.3	1.5	1.4

5.2.2. Sum of Squares of Main Effect (P, S and Q)

Sum of Square of Power Potential, SSP

$$= [1/27(57.6^2+46.2^2+38.2^2)]\text{-C} \quad = 7.05$$

Sum of Square of Scan Speed, SSS

$$= [1/27(49.3^2+46.8^2+45.9^2)]\text{-C} \quad = 0.23$$

Sum of Square of Quench flow rate, SSQ

$$= [1/27(48.9^2+48.1^2+45^2)]\text{-C} \quad = 0.32$$

5.2.3. Two Way Interactions of Sum of Squares (PS, SQ and PQ)

Sum of Square of Power Potential and Scan Speed

$$= [1/9(20.1^2+\ldots 12.9^2)] \text{–C} = 0.52$$

Sum of Square of Scan Speed and Quench flow rate

$$= [1/9(17.1^2+\ldots . 13.8^2)]\text{-C} = 0.91$$

Sum of Square of Power Potential and Quench flow rate

$$= [1/9(21^2+\ldots 12.3^2)]\text{-C} \quad = 0.45$$

5.2.4. Three Way Interactions of Sum of Square (PSQ)

Sum of Square of Power Potential, Scan speed and Quench flow rate

$$= \text{SST}_r - \{\text{SSP-SSS-SSQ-SSPS-SSSQ-SSPQ}\}$$

$$= 12.20\text{-}7.05\text{-}0.238\text{-}0.32\text{-}0.52\text{-}0.91\text{-}0.45$$

$$= 2.712$$

5.2.4.1. Model Calculation (Case Depth- Back of the Bar)

Total sum of the run

$$= (4.3+4.3+4.3+\ldots .1.0+1.1+1.2) = 191.8$$

Number of Treatments
= 3 (3 Factors)

Number of Levels
= 3

Number of replicates (r)
= 3

Total of the observations

under all factor levels
= N = abcr = 3x3x3x3=81

Correction factor, (C)
$= (191.8)^2/81$

= 454.16

Sum of Squares of Treatment, (SST)
$= (4.3^2 + 4.3^2 + 4.3^2 + \ldots 1.0^2 + 1.1^2 + 1.2^2)$

= (535.93)-C

= 81.8

Sum of Squares of Treatment with replicates, (SST_r)
$= 1/3(12.9^2 + 12.3^2 + \ldots 3.3^2)$

= 1/3(1604.44)-C

= 80.65

Sum of Squares of Replicate, (SSR)
$= 1/27(63^2 + 63.5^2 + 65.3^2)$-C

= 0.111

Sum of Squares of Error, (SSE)
$= SST - SST_r - SSR$

= 81.8-80.65-0.111

= 1.039

5.3.1. Sum of Squares of Main effect (P, S and Q)

Sum of Square of Power Potential, SSP
$= [1/27(84.6^2 + 71.1^2 + 36.1^2)]$-C = 46.417

Sum of Square of Scan Speed, SSS
$= [1/27(72.1^2 + 55.8^2 + 63.9^2)]$-C = 4.923

Sum of Square of Quench flow rate, SSQ
$= [1/27(67.9^2 + 64.5^2 + 59.4)]$-C =1.359

5.3.2. Two Way Interactions of Sum of Squares (PS, SQ and PQ)

Sum of Square of Power Potential and Scan Speed
$= [1/9(35.1^2 + \ldots 10.8^2)]$ –C =14.19

Sum of Square of Scan Speed and Quench flow rate
$= [1/9(27.4^2 + \ldots .18.9^2)]$-C = 4.65

<table>
<tr><td></td><td colspan="3">S</td><td></td><td colspan="3">Q</td><td></td><td colspan="3">Q</td></tr>
<tr><td>84.6</td><td>35.1</td><td>18.6</td><td>30.9</td><td rowspan="3">P</td><td colspan="3" rowspan="3"></td><td>72.1</td><td>27.4</td><td>25.2</td><td>19.5</td></tr>
<tr><td>71.1</td><td>22.2</td><td>26.7</td><td>22.2</td><td>55.8</td><td>16.8</td><td>18.0</td><td>21.0</td></tr>
<tr><td>36.1</td><td>14.8</td><td>10.5</td><td>10.8</td><td>63.9</td><td>23.7</td><td>21.3</td><td>18.9</td></tr>
<tr><td>191.8</td><td>72.1</td><td>55.8</td><td>63.9</td><td>84.6</td><td>28.2</td><td>29.4</td><td>27.0</td><td>191.8</td><td>67.9</td><td>64.5</td><td>59.4</td></tr>
<tr><td rowspan="4">P</td><td colspan="3" rowspan="4"></td><td>71.1</td><td>25.5</td><td>23.7</td><td>21.9</td><td rowspan="4">S</td><td colspan="3" rowspan="4"></td></tr>
<tr><td>36.1</td><td>14.2</td><td>11.4</td><td>10.5</td></tr>
<tr><td>191.8</td><td>67.9</td><td>64.5</td><td>59.4</td></tr>
<tr><td></td><td colspan="3"></td></tr>
</table>

Table 20. ANOVA with F-Test Material: AISI 4340 (Case Depth Below the Teeth)

Variable	Sum of Squares	Degrees of Freedom	Mean Square	F	Significant Ranking
Replicates	0.013	2	0.0065	0.810	-
P	7.05	2	3.525	439.526	1
S	0.238	2	0.119	14.837	6
Q	0.320	2	0.160	19.950	4
PS	0.520	4	0.130	16.209	5
SQ	0.910	4	0.2275	28.366	3
PQ	0.450	4	0.1125	14.027	7
PSQ	2.712	8	0.339	42.269	2
ERROR	0.417	52	0.00802	-	
TOTAL	-	80			

Sum of Square of Power Potential and
Quench flow rate = $[1/9(28.2^2+\ldots 10.5^2)]-C = 0.508$

5.3.3. Three Way Interactions of Sum of Square (PSQ)

Sum of Square of Power Potential, Scan speed and Quench flow rate

$= SST_r$ - {SSP-SSS-SSQ-SSPS-SSSQ-SSPQ}

= 80.65-46.417-4.923-1.359-14.19-4.65-0.508

= 8.603

Regression analysis is done using MATLAB and the Regression equations (Equation to predict the Case depth – (Below the teeth and Back of the bar) of the material AISI 4340) are found and given below.

AISI 4340

```
Coeff =

1.0000 5.5000 1.3400 15.0000     2.5     4.3
1.0000 5.5000 1.3400 17.5000     2.3     4.1
1.0000 5.5000 1.3400 20.0000     1.9     3.3
1.0000 5.5000 1.7200 15.0000     2.4     2.8
1.0000 5.5000 1.7200 17.5000     2.0     1.1
1.0000 5.5000 1.7200 20.0000     2.3     2.3
1.0000 5.5000 2.1400 15.0000     2.5     4.0
1.0000 5.5000 2.1400 17.5000     1.5     2.9
1.0000 5.5000 2.1400 20.0000     1.8     3.6
1.0000 7.0500 1.3400 15.0000     1.9     2.7
1.0000 7.0500 1.3400 17.5000     1.5     2.7
1.0000 7.0500 1.3400 20.0000     2.0     2.0
1.0000 7.0500 1.7200 15.0000     1.5     3.1
1.0000 7.0500 1.7200 17.5000     1.7     2.3
1.0000 7.0500 1.7200 20.0000     1.6     3.5
1.0000 7.0500 2.1400 15.0000     1.8     2.7
1.0000 7.0500 2.1400 17.5000     1.4     2.9
1.0000 7.0500 2.1400 20.0000     1.9     1.8
1.0000 8.5000 1.3400 15.0000     2.1     1.8
1.0000 8.5000 1.3400 17.5000     1.3     1.6
1.0000 8.5000 1.3400 20.0000     1.3     1.2
1.0000 8.5000 1.7200 15.0000     1.4     1.4
1.0000 8.5000 1.7200 17.5000     1.3     0.9
1.0000 8.5000 1.7200 20.0000     1.3     1.2
1.0000 8.5000 2.1400 15.0000     1.5     1.2
1.0000 8.5000 2.1400 17.5000     1.3     1.3
1.0000 8.5000 2.1400 20.0000     1.5     1.2
```

The coefficients for the formation of case depth (below the teeth and back of the bar) equation are,

4.3660 8.0291

-0.2265 -0.6135

-0.2127 -0.2797

-0.0367 -0.0500

Equations to predict the Case depth (below the teeth and back of the bar) of the material AISI 4340 are given under.

$$Y_{BT} = 4.3660-0.2265P-0.2127S-0.0367Q \quad (6)$$

$$Y_{BB} = 8.0291-0.6135P-0.2797S-0.0500Q \quad (7)$$

Similarly equation to predict the Case depth (below the teeth and back of the bar) of the material AISI 1055

$$Y_{BT} = 3.4984-0.1683P-0.2474S-0.0111Q \quad (8)$$

$$Y_{BB} = 10.0772-0.7195P-0.7582S-0.0600Q \quad (9)$$

6. RESULTS AND DISCUSSION

6.1. Optimization of Carburizing Process Variables

The present research is concerned with the optimization of process variables and identification of the root cause for the inconsistency in hardness and case depth and distortion in Gas carburized materials, e.g., pinion. After holding extensive consultation with the personnels of all the departments in the industry in which this research has been carried out, it is concluded that preheating, carbon potential, holding position, furnace temperature, carburising time, quenching medium, quenching temperature, quenching time, tempering temperature and tempering time are the influential variables responsible for the surface integrity of the components. Based on this a Cause and Effect Analysis is made and Shewerts' diagram (Fig. **12**) is drawn.

The optimization result (Table **7**) and Response graphs (Figs. **4e**, **6e**) indicates that preheating the material before subjecting to Gas carburizing process improves the hardness and case depth. Even though, this process is employed to relieve the internal stresses, no remarkable microstructural changes occur during this process. Internal stresses are

Table 21. ANOVA with F-Test Material: AISI 4340 (Case Depth Back of the Bar)

Variable	Sum of Squares	Degrees of Freedom	Mean Square	F	Significant Ranking
Replicates	0.111	2	0.0555	2.788	
P	46.417	2	23.2085	1166.256	1
S	4.923	2	2.4615	123.693	3
Q	1.359	2	0.6795	34.145	6
PS	14.190	4	3.5475	178.266	2
SQ	4.650	4	1.1625	58.417	4
PQ	0.508	4	0.127	6.3819	7
PSQ	8.603	8	1.075	54.020	5
ERROR	1.039	52	0.0199	-	
TOTAL	-	80			

Table 22. ANOVA with F-Test - Material: AISI 1055 (Case Depth Below the Teeth)

Variable	Sum of Squares	Degrees of Freedom	Mean Square	F	Significant Ranking
Replicates	0.0274	2	0.0137	1.201	
P	3.340	2	1.60	140.350	1
S	0.770	2	0.385	33.772	2
Q	0.247	2	0.1235	10.833	6
PS	0.850	4	0.2125	18.640	5
SQ	1.080	4	0.270	23.684	3
PQ	0.890	4	0.2225	19.517	4
PSQ	0.860	8	0.1075	9.423	7
ERROR	0.5926	52	0.0114	-	
TOTAL	-	80			

Table 23. ANOVA with F-Test - Material: AISI 1055 (Case Depth Back of the Bar)

Variable	Sum of Squares	Degrees of Freedom	Mean Square	F	Significant Ranking
Replicates	0.0274	2	0.0137	1.507	-
P	70.80	2	35.40	3895.246	1
S	5.286	2	2.643	290.823	2
Q	2.286	2	1.143	125.770	4
PS	1.194	4	0.2985	32.8455	7
SQ	1.768	4	0.442	48.635	5
PQ	5.334	4	1.3335	146.73	3
PSQ	2.432	8	0.304	33.450	6
ERROR	0.4726	52	0.009088	-	
TOTAL	-	80			

developed during machining and grinding. This pre-carburizing process removes these stresses. It holds well with the remarks given by *Shen-Chih Lee and Wei -Youe Ho (1989)* in their paper *"The effects of surface hardening on Fracture Toughness of Carburized steel"* [22]. Further, it is observed that the extent to which the stresses can be relieved depends on the temperature employed, holding time and uniformity in cooling.

Tables **6** and **11** shows that furnace temperature is having a significant effect on obtainable hardness and case depth. The reason for this may be given as below. At higher furnace temperature, formation of water vapour is less. Water vapour is a strongly decarburizing gas but whether this decarburizing tendency will actually reveal itself in practice depends on a number of factors. The first is the concentration in which the water vapour is present and the second is the nature of the carburizing gases in particular gas mixture under consideration. There is perhaps, more contradictory evidence on this subject of the effect of water vapour in gas carburizing and in the heat treatment of steels than any other single item and it is quite clear that a lot more work remains to be carried out before an absolutely clear picture is obtained. It is possible that in small amount, water vapour actually has a beneficial effect on carburizing, an effect which seems to be catalytic in nature. However, the present study shows that higher furnace temperature (940°C) gives high hardness and case depth.

In the present analysis, Optimum Gas Carburising Process conditions to obtain higher surface hardness with more case depth are given in Table **24**.

Analysis of variance is done for EN 29 and EN 34. The ANOVA results (Tables **6** and **11**) and optimum conditions for hardness and Case depth (Table **7** – Response graph and S/N ratio) indicate that the interaction between Furnace temperature and quenching time is having 25 - 30% influence on the hardness and case depth. Further, the present optimization analysis shows that Signal to noise ratio method has also given the same optimal variable levels/best treatment combination levels with the Response Graph analysis.

To check the optimum results obtained through Taguchis' DOE, confirmation trials are carried out and the results are tabulated in Table **25**. From the table it is clear that the predicted conditions for higher hardness and case depth suits well with the experimental results.

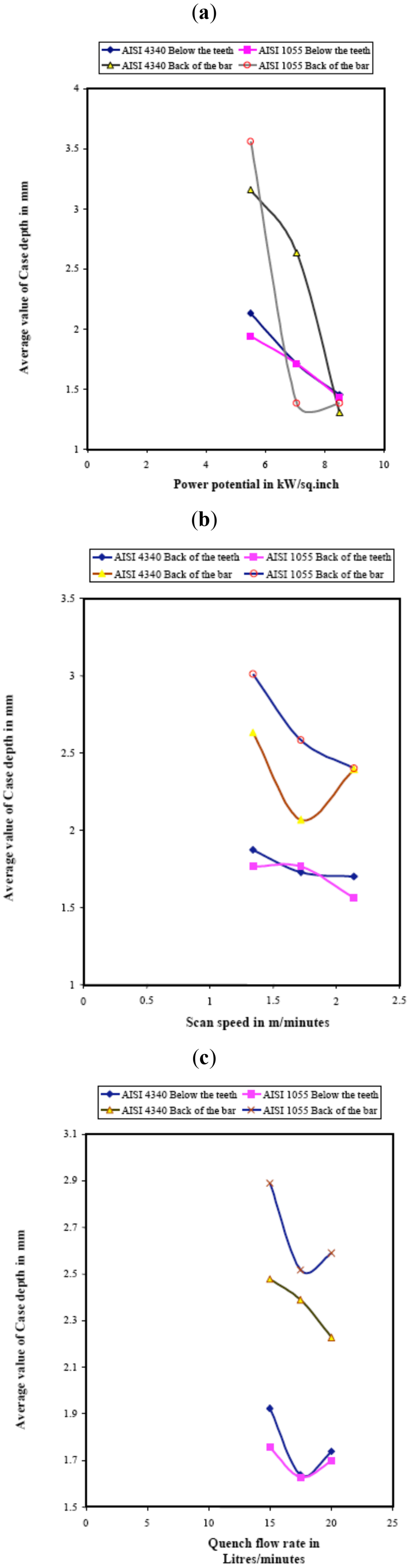


Fig. (11). **(a-c)** Process variables *vs* Case depth.

EN 34 (Nickel Molybdenum alloy steel) are characterized by higher tensile strength and toughness values, improved fatigue strength, impact resistance and shear strength. EN 29 (Chromium Molybdenum alloy steel) steels are generally case carburized. Increasing Chromium content increases the wear resistance of case. However, toughness in the core is somewhat reduced by increasing Chromium content. The experimental values (EN 29: Maximum case depth -1 mm and Maximum hardness – 81.5 HRA, EN 34: Maximum case depth – 0.9625mm and Maximum Hardness – 80.5 HRA) on case depth and Hardness shows that Chromium Molybdenum steel is giving a slightly higher hardness and case depth than the Nickel Molybdenum alloy steel.

It is because of presence of more amount of Molybdenum in EN 29. The addition of Molybdenum improves hardenability, ductility, toughness, and elevated temperature properties of the steel and also Molybdenum inhibits grain growth and makes the steel less susceptible to temper brittleness.

6.2. Optimization of Induction Hardening Process Variables

Cause and Effect Analysis is made and Shewerts' diagram (Fig. **13**) is drawn for the Induction hardening process. From this, the major contributing process variables have been identified.

Analysis of variance is done for AISI 4340, AISI 1055. The ANOVA results, and F-Test results (Tables **16**, **17**, **20-23**) shows that Power potential has more influence on the hardness and case depth of the Induction hardened components. *This matches with the suggestion given by Mehmet Cengiz Kayacan and Oguz Colak 2004.* In order to obtain the significance and effect of each factor and their interaction, the sum of the squares, Degrees of freedom, Mean square and F are calculated first. Based on these calculations the Ranking and significance of each variable are done [23-24]. F-Test ranking also shows that Power potential is the number one variable having effect on surface hardness and case depth [25].

Figs. (**9a-c**, **11a-c**) shows that under optimal conditions (Power potential 5.5 kW/inch^2, Scan speed 1.72 m/minutes and Quench flow rate 15 litres/min) the hardness and case depth is maximum for the materials AISI 4340 and AISI 1055 with low distortion.

In the present experimental analysis, optimum induction hardening conditions to obtain high hardness and with low distortion is given in Table **26**.

Regression analysis is done and the controlling equation to predict the Hardness and Case depth of Induction hardened components at any parametric conditions has been developed [26]. To check the Regression equations confirmation trials are carried out and the results are tabulated in Table **27**. It shows that there is a good match between the experimental results and predicted regression results.

7. CONCLUDING REMARKS

- Furnace Temperature and Quenching time have equal influence on the Surface integrity of the case hardened components in Gas Carburizing. The investigation reveals that the interaction effect

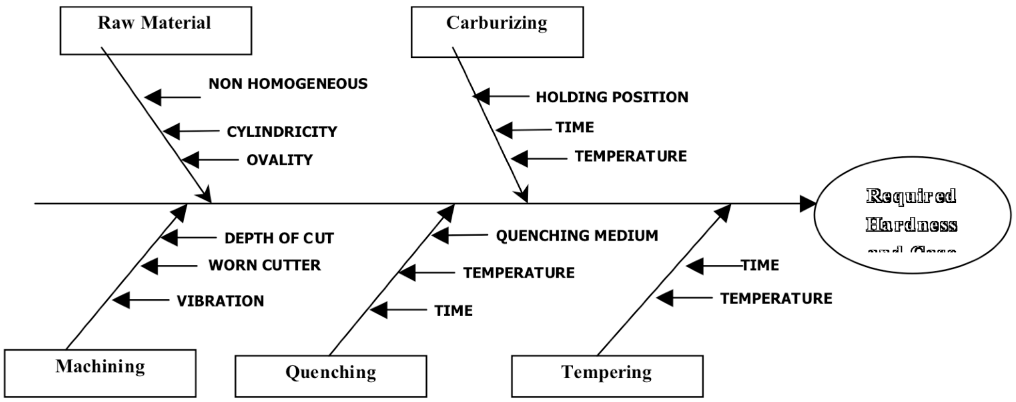


Fig. (12). Shewerts' diagram for Gas carburizing process.

between Furnace Temperature and Quenching time is 30%.

- Optimum Gas Carburizing Process conditions as per the present test results to obtain Higher surface hardness with more case depth are,

Table 24. Optimum Gas Carburising Process Conditions

S. No.	Process Variables	Values with Unit
01	Furnace Temperature	940°C
02	Quenching Time	90 Minutes
03	Tempering Temperature	200°C
04	Tempering Time	120 Minutes

Table 25. Experimental Trials *vs* Predicted Values by Taguchis' DOE

Experimental Values				Predicted Values by Calculation (Optimum Conditions Values)			
EN 29		EN 34		EN 29		EN 34	
Hardness HRA	Case Depth mm	Hardness HRA	Case Depth mm	Hardness HRA	Case Depth mm	Hardness HRA	Case Depth mm
81	0.90	79	0.80	**81.5336**	**0.96265**	**80.7802**	**0.89454**
80	0.90	80	0.90				
81	1.00	79	0.80				
80	1.00	80	0.80				
81	0.90	80	0.90				

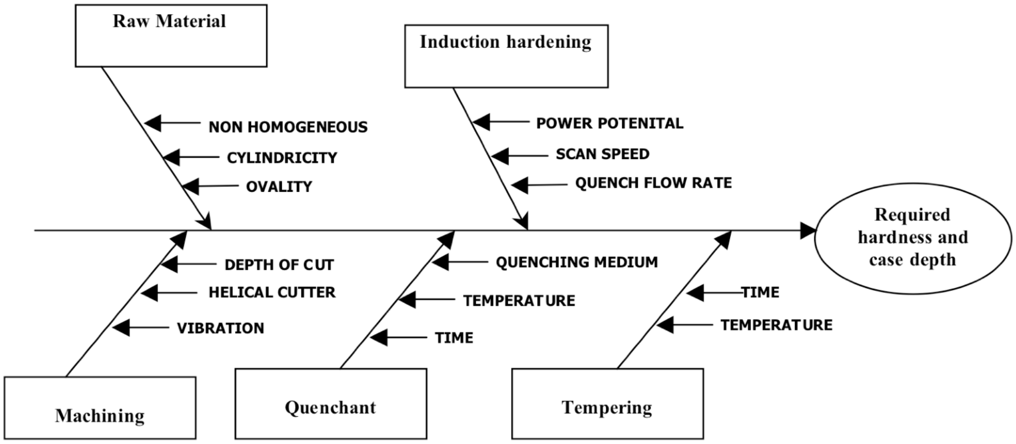


Fig. (13). Shewerts' diagram for Induction hardening process.

Table 26. Optimum Induction Hardening Process Conditions

S. No.	Process Variables	Values with Unit
01	Power Potential	5.5kW/ inch^2
02	Scan Speed	1.72 m/minutes
03	Quench Flow rate	15 Litres/minutes
04	Frequency	10kHz

- In the present experimental analysis, optimum induction hardening conditions to obtain high hardness and with low distortion are,
- The Controlling equations developed through Regression Analysis are useful in fixing the parameters at the required level.

ACKNOWLEDGEMENTS

The authors are thankful to Rane (Madras) Pvt. Ltd., Thirubhuvanai, Pondicherry and IGCAR Kalpakkam for providing the experimental, testing and measurement facilities.

Table 27. Confirmation Trials Results *vs* Predicted Optimum Results

Trials	Experimental Conditions			Hardness in HRA		Distortion in mm		Case Depth Below the Teeth in mm		Case Depth Back of the Bar in mm	
	P	S	Q	a	b	a	b	a	b	a	b
01	4.5	1.30	10.0	87.0	85.0	2.04	2.50	2.70	2.5	4.40	4.40
02	5.0	1.5	12.5	83.7	82.0	1.74	1.95	2.45	2.2	3.91	3.50
03	**5.5**	**1.72**	**15.0**	**80.4**	**79.0**	**1.43**	**1.50**	**2.20**	**2.0**	**3.42**	**3.30**
04	6.0	2.0	18.0	76.8	76.0	1.08	1.00	1.92	1.8	2.88	2.50

a – Predicted values from Regression equation; b – Confirmation trials values.

Table 7.1. Optimum Conditions for Gas Carburizing Process

S. No.	Process Variables	Values with Unit
01	Furnace Temperature	940°C
02	Quenching Time	90 Minutes
03	Tempering Temperature	200°C
04	Tempering Time	120 Minutes

Table 7.2. Optimum Conditions for Induction Hardening Process

S. No.	Process Variables	Values with Unit
01	Power Potential	5.5kW/ inch^2
02	Scan Speed	1.72 m/minutes
03	Quench Flow rate	15 Litres/minutes
04	Frequency	10kHz

- Preheating before Gas Carburizing increases the obtainable hardness and case depth of the material. It holds good with the remarks given by Shen-*Chih Lee and Wei -Youe Ho(1989)* on their paper *"The effects of surface hardening on Fracture Toughness of Carburized steel"*.
- Power Potential has major influence on the Surface Hardness and Case depth of the Induction hardened components. *This matches with the suggestion given by Mehmet Cengiz Kayacan and Oguz Colak 2004* [27].

REFERENCES

[1] Alford LP, Beatty HR. Principles of industrial management. Ronald P: US 1951.

[2] Grossmann MA, Bain EC. Principles of heat treatment. Hetenyi M, Ed. ASM. Ohio: Cleveland 1964.

[3] American Iron and Steel Institute. Steel products manual tool steels 1976.

[4] Schimizu N, Tamura I. An examination of the relation between quench-hardening behaviour of steel and cooling curve in oil. Trans ISIJ 1978; 18: 445-50.

[5] Barbacki A, Mikolajski E. Optimization of heat treatment conditions for maximum toughness of high strength silicon steel. Int J Mater Process Technol 1998; 78: 18-23.

[6] Fee AR, Robert, Edward SL. Mechanical testing of materials. ASMHandbook 1995; vol. 8: pp. 91-3.

[7] Arkhipov YAJ, Batyrev VA, Polotskii MS. internal oxidation during carburizing and heat treating. Metals Trans A 1972; 9A(11): 1553-60.

[8] ASM metals handbook: heat treating. 9th ed. Ohio: Metal Park 1981; vol. 4.

[9] Montgomery DC. Introduction to statistical quality control. Singapore: John Wiley and Sons (ASIA) Pte Ltd. 2001.

[10] Singh H, Kumar P. Tool wear optimization in turning operation by Taguchi method. Indian J Eng Mater Sci 2004: 11; 19-24

[11] Harris FE. Case Depth – An attempt at practical definition. Metal Prog 1943.

[12] Rajan TV, Sharma CP, Sharma A. Heat treatment principles and techniques. 8th ed. India: Prentice-Hall of (India) Pvt. Ltd. 1985.

[13] Brooks CR. Principles of heat treatment of plain carbon and low alloy steels. ASM International, Ohio: Metal Park 1996.

[14] Osborn HB. Surface hardening by induction heat. Metal Prog 1955; pp. 105-9.

[15] Scott Mackenzie D. Principles of quenching for induction hardening, presented at the SME induction hardening seminar, Michigan, 15th May 2002, Nov 2002; pp.1-10.

[16] Bodart O, Boureau AV, Touzani R. Numerical investigation of optimal control of induction heating processes. Int J Appl Math Model 2001; 25: 697-712.

[17] Dughiero F, Battistetti M. Optimization procedures in the design of continuous induction hardening and tempering installations for magnetic steel bars. IEEE Trans Magnet 1998; 34: 2865-8.

[18] Ashby MF, Easterling KE. The transformation hardening of steel surfaces by laser beams –I Hypo –eutectoid Steels. Acta Metall 1984; 32(A11): 1935-48.

[19] Averbach BL, Cohen, Fletcher's. The dimensional stability of steel -part-iii, decomposition of martensite and austenite at room temperature. Trans ASM 1948; 40: 726-8.

[20] Fischer FD. Simplified calculation of temperature filed in heat treated cylinder using temperature measured at one point. Mater Sci Technol 1992; 1; 468-73.

[21] Iozinskii MG. Industrial application of induction heating. Oxford Pregamon Press Ltd 1969.

[22] Lee SC, Ho WY. The effects of surface hardening on fracture toughness of carburized steel. Metall Trans A 1989; 20A: 519-24.

[23] Matsui K, Hata H, Kadogawa H, Yoshiyuki K. Research on practical application of dual frequency induction hardening to gears. Int J Soc Automot Eng 1998; 19: 351-71.

[24] Kayacan MC. Design and construction of a set-up for induction hardening, M. Sc., Thesis, University of Gaziantep 1991.

[25] Shary B, Osborn Jr. Surface hardening by induction. Cleveland: Park-Ohio Industries Inc. 1974: 181-3.

[26] Semiatin SL, Stutz DE. Industrial heat treatment of Steel 1986.

[27] Kayacan MC, Colak O. A fuzzy approach for induction hardening parameters selection. Int J Mater Des 2004: 25; 155-61.

Received: October 6, 2009 Revised: October 8, 2009 Accepted: October 27, 2009